THE OHIO STATE UNIVERSITY

IEEE ISIT 2016

Optimizing Data Freshness, Throughput, and Delay in Multi-Server Information-Update Systems

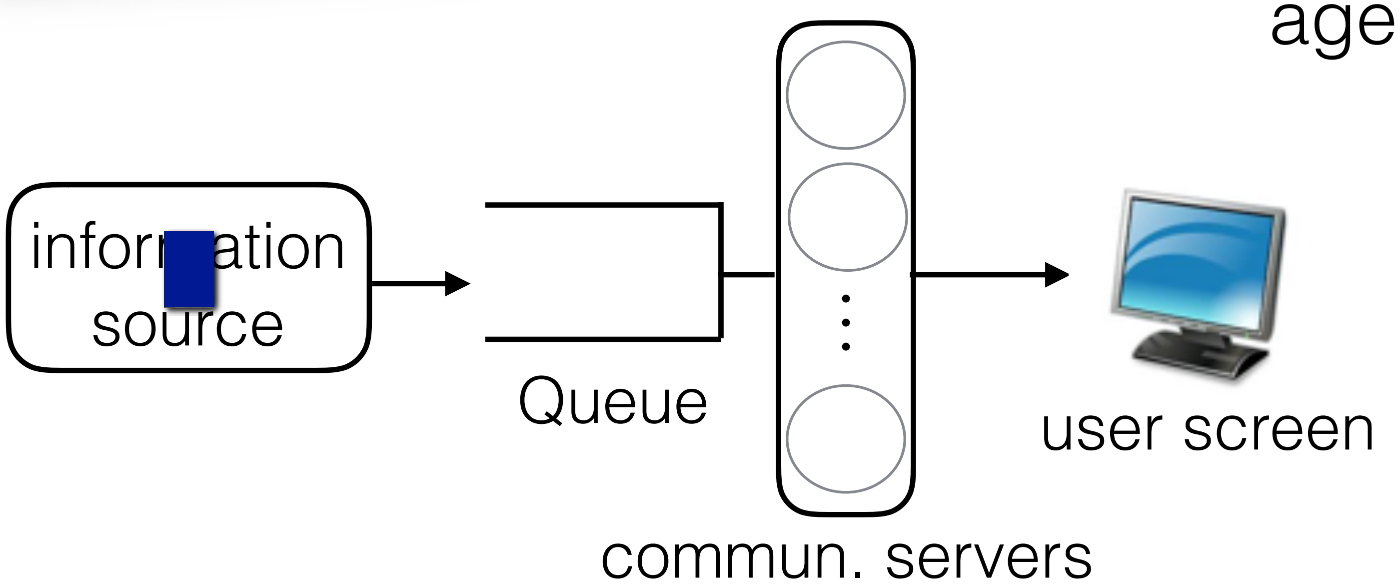
Ahmed M. Bedewy

Joint work with Yin Sun, Ness B. Shroff

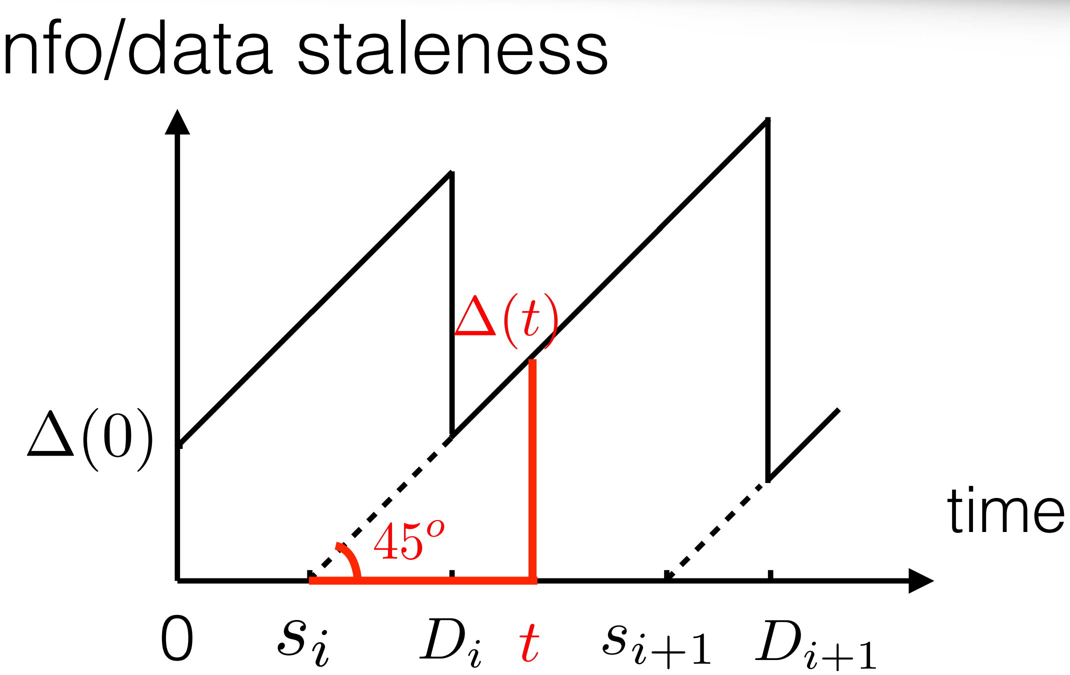
Departments of ECE,
The Ohio State University

Nov. 7th 2016

What is the Age of Information?



Information-update system

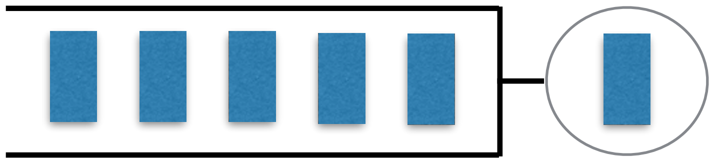


- A stream of messages generated at an information source
- To be sent to a destination via **multiple** communication channels/servers
- Update i is generated at time s_i and delivered at time D_i

Definition: at any time t , **the age-of-information** $\Delta(t)$ is the "age" of the freshest message available at the destination

$$\Delta(t) = t - \max\{s_i : D_i \le t\}$$

Difference between Delay & Age



High arrival rate

Low arrival rate

Age is different from delay. [Kaul, Yates, Gruteser, Infocom 2012]

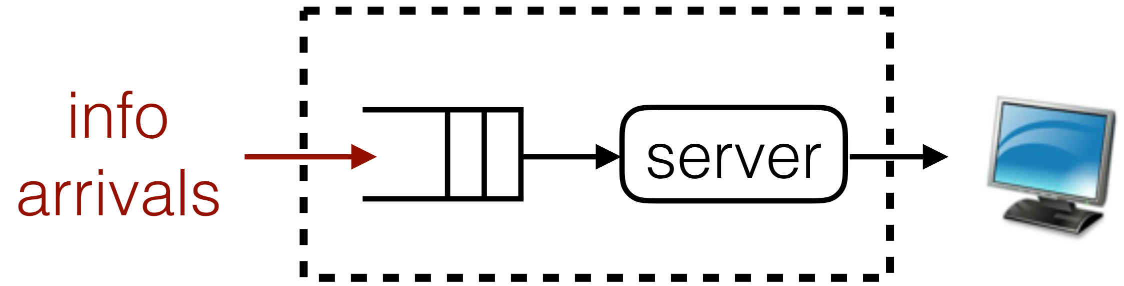
- High arrival rate: long waiting time, packets becomes stale
- Low arrival rate: short queue, but infrequent updates

High age

Age is not monotonic wrt queue length

- For delay, Little's Law says: $L = \lambda W$
- Delay grows linearly wrt queue length

Open Questions



Enqueue-and-forward model

- **Age Characterization and Age Reduction:**
  - M/M/1, M/D/1, D/M/1 [Kaul, Yates, Gruteser 2012]
  - M/M/2 [Kam, Kompella, Ephremides 2014]
  - Multi-sources [Yates, Kaul ISIT 2012] [Huang, Modiano 2015]
  - Packet management, LCFS [Kam, Kompella, Ephremides 2013, 2014]
  - Channel state info [Costa, Valentin, Ephremides, 2015]
  - LCFS (single server) with & without preemption [Kaul, Yates, Gruteser 2012]

- **Question 1:** Which policy is **age-optimal**?

Open Questions (cont.)

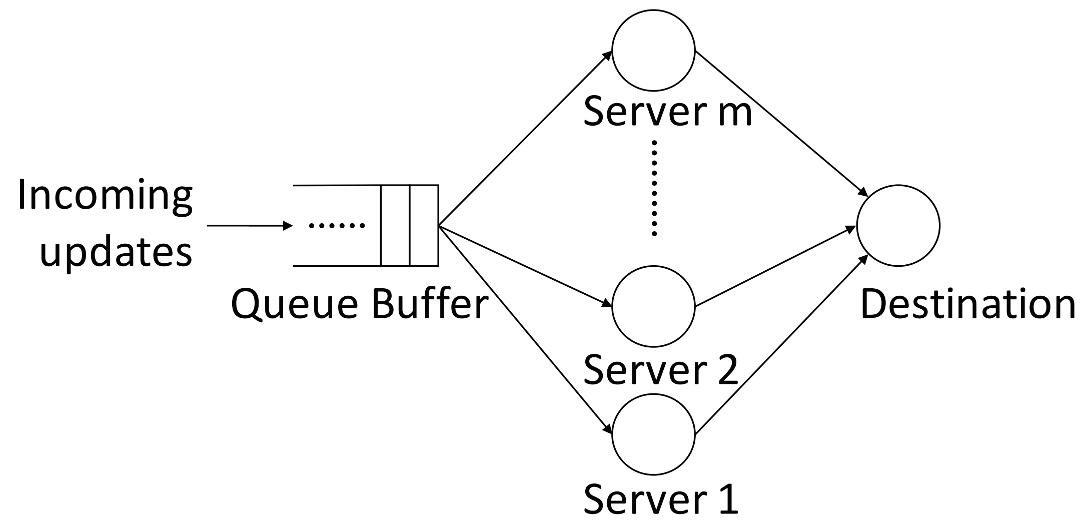
- **Information Updates**
  - News, emails, notifications, stock quotes, …

- Users may be interested in not just the latest updates, but also past news.

- **Question 2**: Is there a policy which simultaneously optimizes **age**, **throughput**, and **delay**?

  - This presentation will answer these two questions

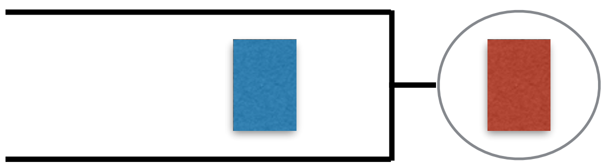
System Model



- An information-update system with
  - **m** *i.i.d.* servers
  - Queue with buffer size B
- Packet i is generated at time s_i ,and arrives at time a_i $(s_1 \leq s_2 \leq \ldots)$
  - **Arbitrary** arrival process (including **non-stationary**)
  - Update packets can arrive **out of order** (e.g., $a_i > a_{i+1}$ but $s_i < s_{i+1}$)
- The set of all causal policies is denoted by Π

Definitions

- **Definition. Service Preemption**: At any time
  - A server can switch to send any packet
  - The preempted packet will be stored back into the queue
  - To be sent at a later time when the servers are available again.



- **Definition. Stochastic Ordering**: Let X and Y be two random variables. Then, X is said to be **stochastically smaller** than Y (denoted as $X \leq_{\mathrm{st}} Y$), if

$$\mathbb{P}\{X > x\} \leq \mathbb{P}\{Y > x\}, \quad \forall x \in \mathbb{R}.$$

- **In other words**: $X \leq_{\mathrm{st}} Y$ **iff** there exist two random variable $\hat{X}$ and $\hat{Y}$, defined on the same probability space, such that

$$\hat{Y} =_{\mathrm{st}} Y \quad \& \quad \hat{X} =_{\mathrm{st}} X$$

and

$$\mathbb{P}\{\hat{X} \leq \hat{Y}\} = 1$$

(here $=_{\mathrm{st}}$ denotes equality in law)

Definitions

- **Definition. Stochastic Ordering of Stochastic Processes:** Two random processes $\{X(t), t \in [0,\infty)\}$ and $\{Y(t), t \in [0,\infty)\}$ satisfies $\{X(t), t \in [0,\infty)\} \leq_{\text{st}} \{Y(t), t \in [0,\infty)\}$ **iff** there exist two random processes $\{\hat{X}(t), t \in [0,\infty)\}$ and $\{\hat{Y}(t), t \in [0,\infty)\}$, defined on the same probability space, such that

$$\{\hat{Y}(t), t \in [0,\infty)\} =_{\text{st}} \{Y(t), t \in [0,\infty)\} \;\;\&\;\; \{\hat{X}(t), t \in [0,\infty)\} =_{\text{st}} \{X(t), t \in [0,\infty)\}$$

and

$$\mathbb{P}[\hat{X}(t) \leq \hat{Y}(t), t \in [0,\infty)] = 1$$

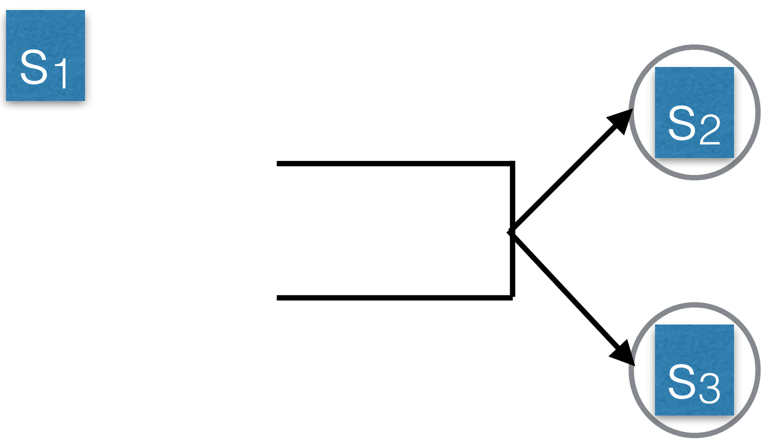
- **Definition. Age Optimality:** A policy $\gamma \in \Pi$ is said to be age-optimal, if for all $\pi \in \Pi$

$$\{\Delta_\gamma(t), t \in [0,\infty)\} \leq_{\text{st}} \{\Delta_\pi(t), t \in [0,\infty)\}.$$

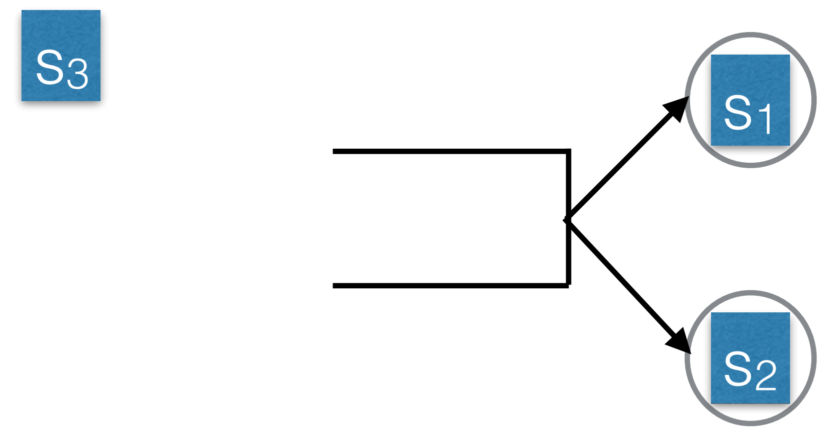
Policy P Algorithm

- Policy P: **Preemptive Last Generated First Served (LGFS)**

- Arrival
  - Stale packet

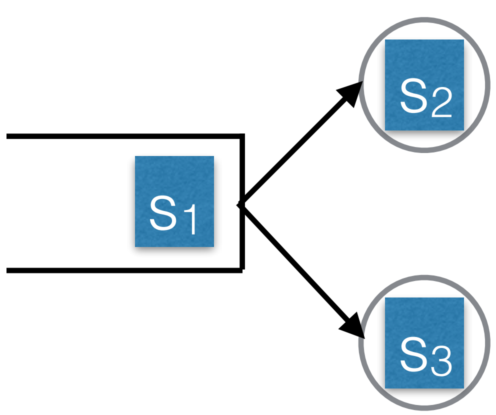


  - Fresh packet



- Policy P is serving the **freshest packets** among all arrived packets.
- Preempted packet is **stored back** in the queue to preserve the **throughput.**

- Departure

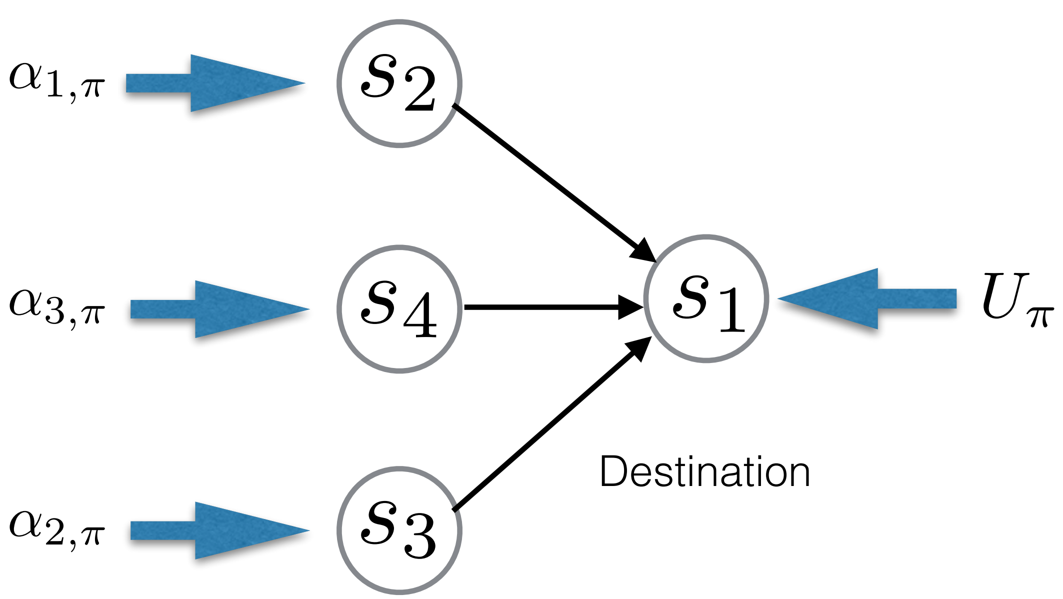


Main Theorem

Theorem 1: For *1. i.i.d. exponential* service time distribution
2. any packet generation and arrival times
3. any buffer sizes B

The **preemptive LGFS** policy is **age-optimal.**

- The system state of policy π is $V_\pi(t) = (U_\pi(t), \alpha_{1,\pi}(t), \ldots, \alpha_{m,\pi}(t))$, where
  - $U_\pi(t)$ is the **largest** time stamp of the **delivered packets**
  - $\alpha_{i,\pi}(t)$ is the *i*-th smallest time stamp of the **packets being transmitted**



3-Servers system

Proof sketch

Policy P vs policy π

$V_\pi(t) = (U_\pi(t), \alpha_{1,\pi}(t), \ldots, \alpha_{m,\pi}(t))$

Step 1: Construction

- $\{\hat{V}_P(t), t \in [0,\infty)\} =_{\text{st}} \{V_P(t), t \in [0,\infty)\}.$
- $\{\hat{V}_\pi(t), t \in [0,\infty)\} =_{\text{st}} \{V_\pi(t), t \in [0,\infty)\}.$

Step 2: Coupling

Policy **P**

$\hat{\alpha}_{m,P}$

$\vdots$

$\hat{\alpha}_{i,P}$

$\vdots$

$\hat{\alpha}_{1,P}$

Policy π

$\hat{\alpha}_{m,\pi}$

$\vdots$

$\hat{\alpha}_{i,\pi}$

$\vdots$

$\hat{\alpha}_{1,\pi}$

Exponential service time distribution is **memory-less**

Step 3: Forward induction in time

- Prove $\mathbb{P}[\hat{V}_P(t) \geq \hat{V}_\pi(t), t \in [0,\infty)] = 1$ $\xrightarrow{\textbf{Def.}}$ $\{V_P(t), t \in [0,\infty)\} \geq_{\text{st}} \{V_\pi(t), t \in [0,\infty)\}. \quad \forall \pi \in \Pi$

$\{\hat{V}_P(t), t \in [0,\infty)\} =_{\text{st}} \{V_P(t), t \in [0,\infty)\}.$

$\{\hat{V}_\pi(t), t \in [0,\infty)\} =_{\text{st}} \{V_\pi(t), t \in [0,\infty)\}.$

$\longrightarrow$ $\{\Delta_P(t), t \in [0,\infty)\} \leq_{\text{st}} \{\Delta_\pi(t), t \in [0,\infty)\}, \ \forall \pi \in \Pi$

Corollaries

- **Corollary**: The age performance of the **preemptive LGFS** policy remains the same for any queue size $B \geq 0$
  - Stale packets are stored back in the queue.

- **Corollary**: For the same system setting as Theorem 1, the preemptive LGFS policy **minimizes**:
  1. The time-average age
  2. Average peak age
  3. Time-average age penalty

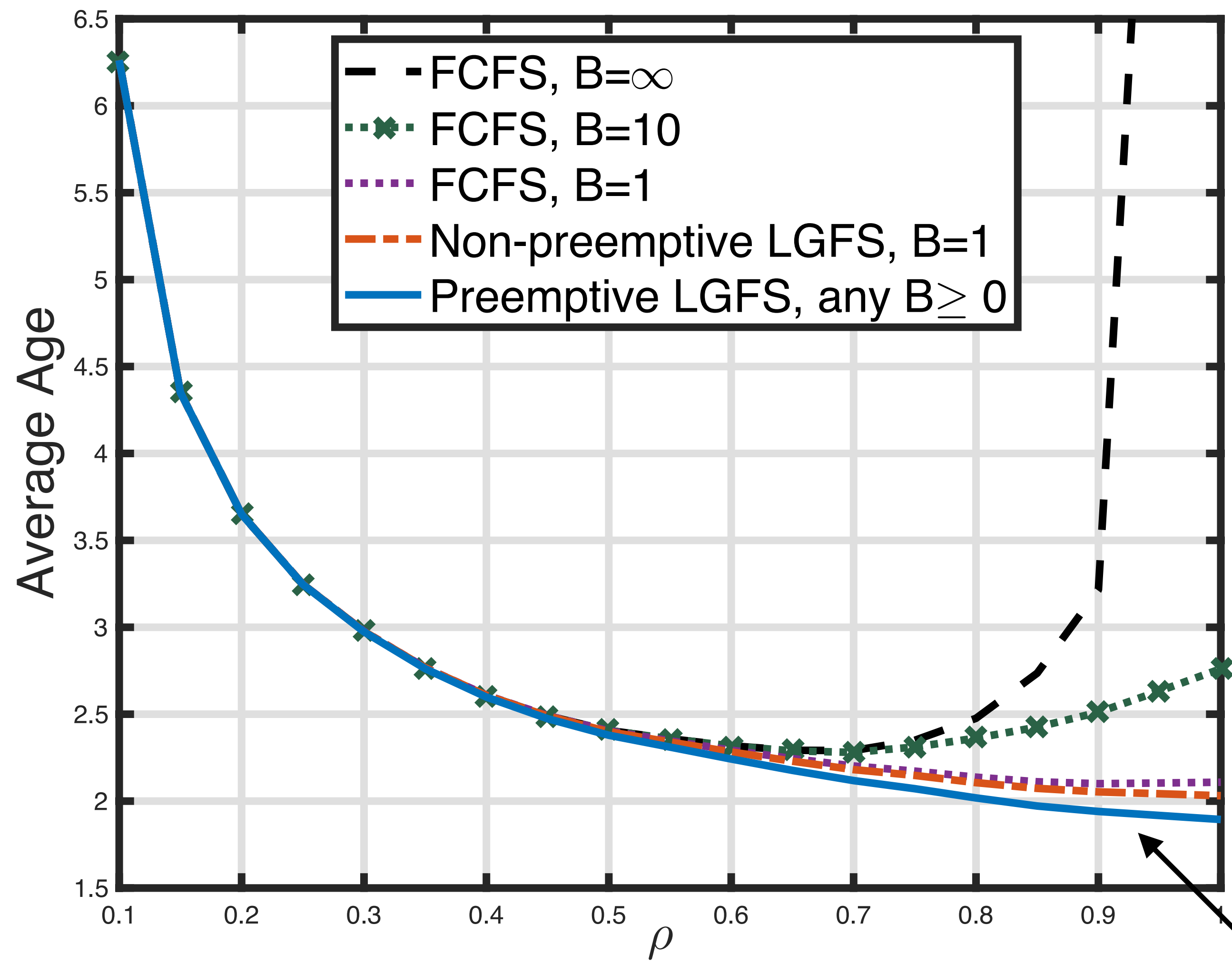
  - These age metrics are increasing function of the age process.

Theorem

Theorem 2: For
1. i.i.d. exponential service time distribution
2. *Any* packet generation and arrival times
3. Infinite buffer size $B = \infty$

The **preemptive LGFS** policy is **throughput-optimal** and **mean-delay-optimal** among all policies in Π .

Simulation Result



Invariant with B

- $m = 5$ servers, buffer size B
- inter-generation times: *i.i.d.* **Erlang-2 distribution**
- $(a_i - s_i)$ is modeled to be either **1** or **100** with **equal probability**

Observations:
1. Preemptive LGFS **outperforms** all other policies.
2. The age performance of the preemptive LGFS is **invariant** for any B
3. **FCFS**: The age gets worse as B increases.

Summary & Future Work

Exponential service time.

- The **preemptive LGFS** optimizes **age**, **throughput** and **delay** among all causally feasible policies for
  - **Arbitrary** packet generation times $s_1, s_2, \ldots$
  - **Arbitrary** arrival process $a_1, a_2, \ldots$ (could be **non-stationary**, **non-ergodic**, **out of order arrivals**)
  - Any number of servers

- Other service time distributions?

Service time dist.	**Exponential**	**NWU Hyperexponential distribution**	**NBU Erlang distribution**
Preemptive LGFS	Age-optimal (any arrival orders)	Not age-optimal in the same policy space	
Non-preemptive LGFS	Near age-optimal		Near age-optimal


Thank You!