THE OHIO STATE UNIVERSITY

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Age-Optimal Information Updates in Multihop Networks

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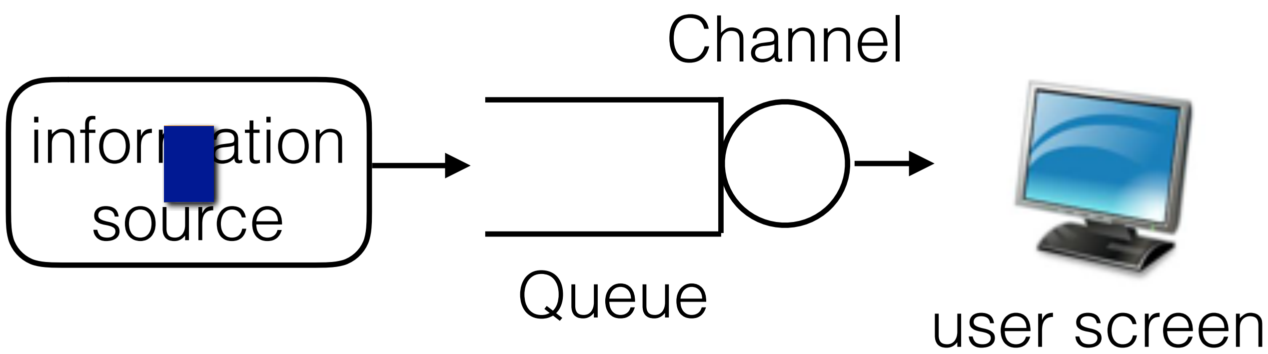
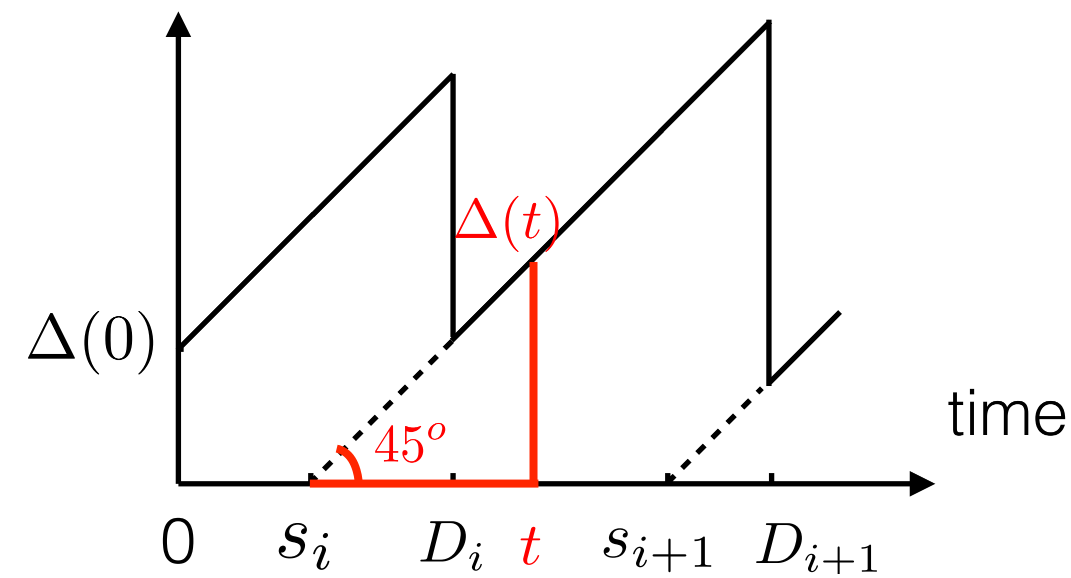
What is the Age of Information?



Information-update system

Age $\Delta(t)$ at receiver



- A stream of messages generated at an information source
- To be sent to a destination via communication channel
- Update i is generated at time s_i and delivered at time D_i

Definition: at time t, the age-of-information $\Delta(t)$ is the “age” of the freshest message available at the destination before time t

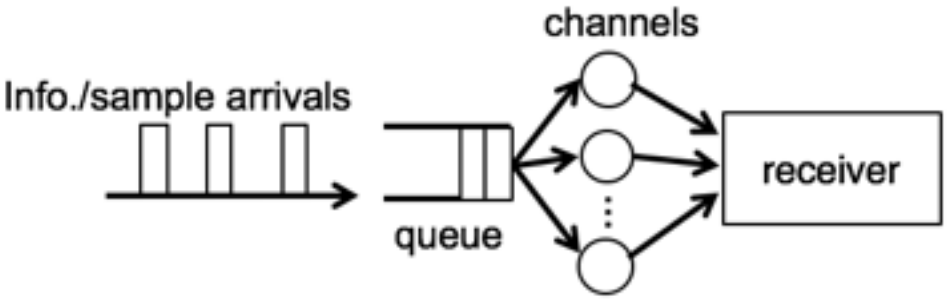
$$\Delta(t) = t - \max\{s_i : D_i \le t\}$$

Motivation

- **Information Updates**
  - News spreading across the Media websites
  - Retweet on Twitter
  - ……

- **Intelligent Transport Systems**
  - Vehicles share information.

- Age-optimality: **Multi-channel** **single hop** network **[Bedewy, Sun, Shroff, ISIT16]**
- **No study** optimized the age in **multi hop network**



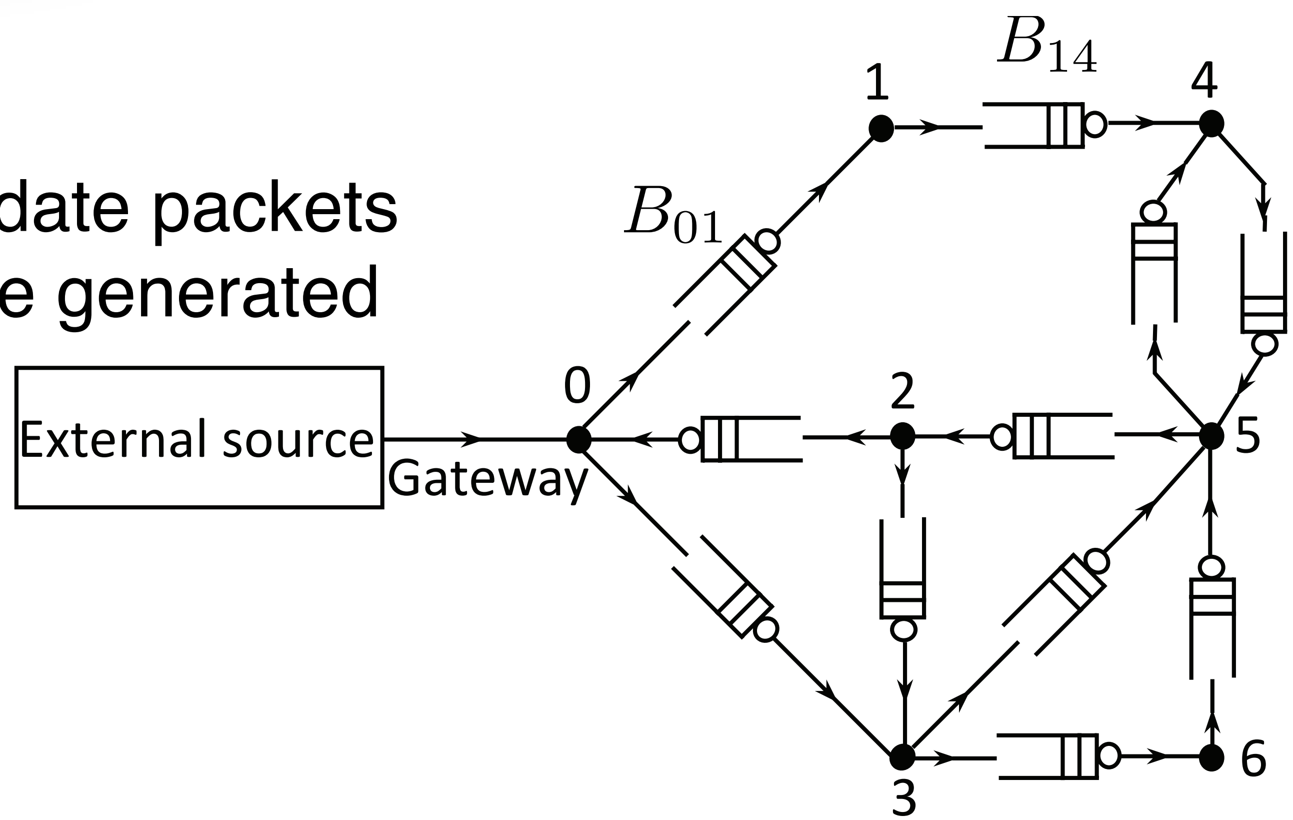
Question

- Can we achieve age-optimality in **general** **multihop networks**?

We will see:
Intuitive policies are age-optimal in a **quite strong** sense.

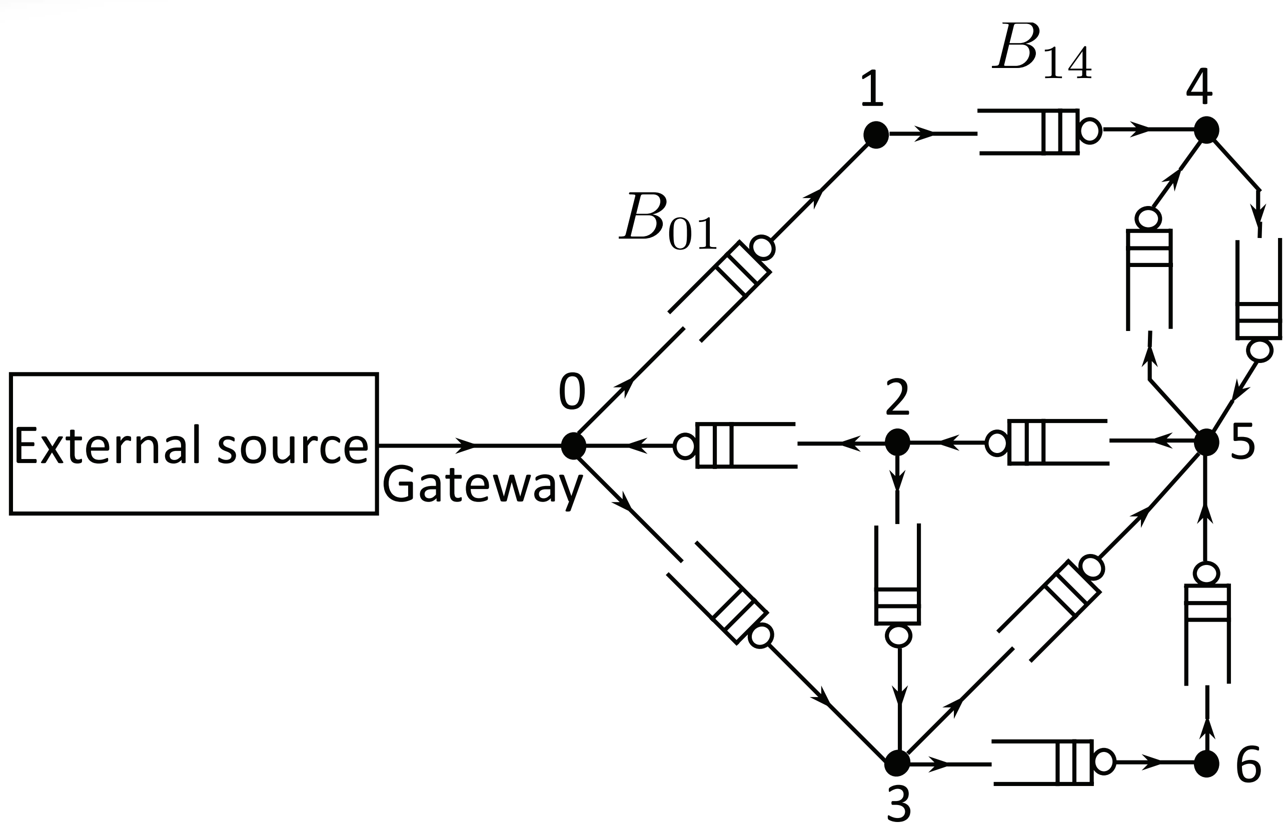
Model: Interference Free Network

n update packets are generated



- **General** multihop Netwerk represented by **directed graph**: $\mathcal{G}(\mathcal{V}, \mathcal{L})$, $|\mathcal{V}| = N$
- **External Arrival** process:
  - Packet i is generated at time s_i, arrives at time a_{i0}. Hence, $s_i \leq a_{i0}$
  - **Arbitrary** packet **generation** & **arrival** processes (could also be **non-stationary**)
  - **Out-of-order** arrivals at node 0 is possible (e.g.,$s_i > s_j$, $a_{i0} < a_{j0}$)
- Packet transmission times are **independent** across links and **i.i.d.** across time

Model: Interference Free Network



- The age at node j is $\Delta_j(t) = t - \max\{s_i : a_{ij} \leq t\}$
- The **age processes of** **all the network nodes** is $\mathbf{\Delta} = \{\Delta_j(t), t \in [0, \infty), j \in \mathcal{V}\}$

We optimize the age processes of all the nodes

General Age Metric

- **Age Penalty Functional** $g(\Delta)$**:** $\boldsymbol{\Delta} = \{\Delta_j(t), t \in [0,\infty), j \in \mathcal{V}\}$
  - **Any non-decreasing functional** g of the **age processes of all nodes** $\boldsymbol{\Delta}$, i.e.,

$$\text{If } \boldsymbol{\Delta}_1 \leq \boldsymbol{\Delta}_2, \text{ then } g(\boldsymbol{\Delta}_1) \leq g(\boldsymbol{\Delta}_2)$$

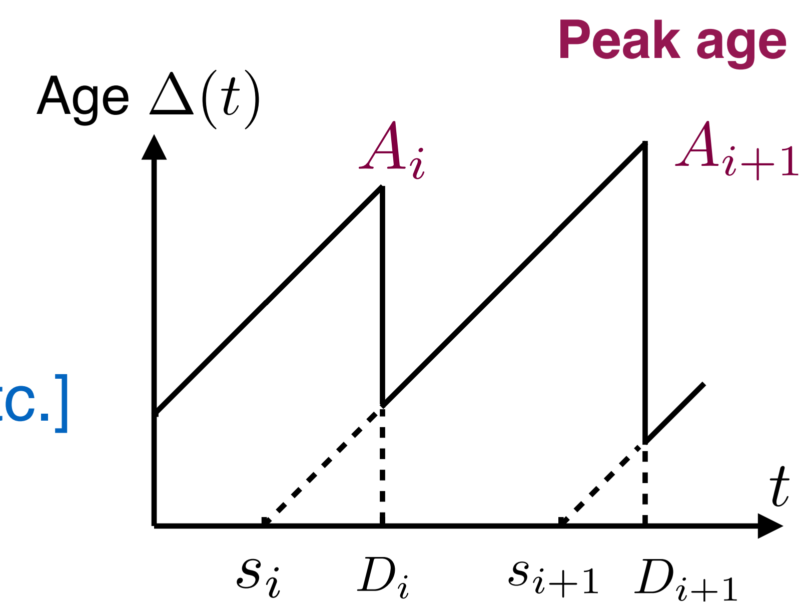
- Prior age metrics as **examples**:

1. *Avg. age:* [Kaul, Yates, Gruteser'12, etc.]

$$g_1(\Delta) = \frac{1}{T}\int_0^T \Delta(t)dt$$

2. *Avg. peak age:* [Costa, Cordreanu, Ephremides' 14, etc.]

$$g_2(\Delta) = \frac{1}{K}\sum_{i=1}^{K} A_i$$



3. *Avg. age penalty function:* [Sun, Uysal, Yates, Koksal, Shroff'16, etc.]

$$g_3(\Delta) = \frac{1}{T}\int_0^T h(\Delta(t))dt$$

(Allow the limits $K, T \to \infty$)

The most general age metric so far.

Age Optimality

- **Definition. Stochastic Ordering**: Let X and Y be two random variables. Then, $X \leq_{\text{st}} Y$

$$\mathbb{P}\{X > x\} \leq \mathbb{P}\{Y > x\}, \quad \forall x \in \mathbb{R}.$$

- A policy γ is said to be **age-optimal** if:
  - Minimizing the **age processes of all nodes** in stochastic ordering sense

$$[\boldsymbol{\Delta}_\gamma | \mathcal{I}] \leq_{\text{st}} [\boldsymbol{\Delta}_\pi | \mathcal{I}] \quad \forall \pi \in \Pi$$

$$\boldsymbol{\Delta} = \{\Delta_j(t), t \in [0, \infty), j \in \mathcal{V}\}$$

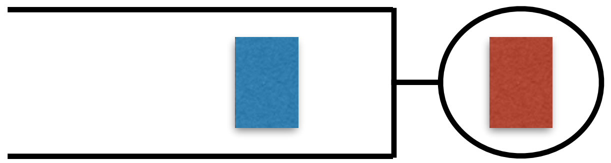
- Equivalently: Minimizing **all** non-decreasing functional of the **age processes of all nodes**

$$\mathbb{E}[g(\boldsymbol{\Delta}_\gamma) | \mathcal{I}] = \min_{\pi \in \Pi} \mathbb{E}[g(\boldsymbol{\Delta}_\pi) | \mathcal{I}]$$

- g: non-decreasing age functional
- $\mathcal{I} = \{n, (s_i, a_{i0})_{i=1}^n, \mathcal{G}(\mathcal{V}, \mathcal{L}), (B_{ij}, (i,j) \in \mathcal{L})\}$: Set of system parameters
- Π: set of causal policies

Scheduling Policies

- **Preemptive Last Generated First Served (prmp-LGFS)** policy:
  - The **last** generated packet is sent **first**
  - When **young** packet arrives
    - **Preempt** **old** packet being transmitted



- **Non-preemptive LGFS (non-prmp-LGFS)**policy:
  - The **last** generated packet is sent **first**
  - Preemption is not allowed
  - After transmission, the link sends the **next** **freshest** packet in its queue

Results for Exponential Service Time

Theorem 1: If packet transmission times are **exponentially distributed**, then for **all** system parameters $\mathcal{I}$ and $\pi \in \Pi$

$$[\boldsymbol{\Delta}_{\text{prmp-LGFS}}|\mathcal{I}] \leq_{\text{st}} [\boldsymbol{\Delta}_{\pi}|\mathcal{I}]$$

or equivalently, for **all** $\mathcal{I}$ and **non-decreasing functional** g

$$\mathbb{E}[g(\boldsymbol{\Delta}_{\text{prmp-LGFS}})|\mathcal{I}] = \min_{\pi\in\Pi} \mathbb{E}[g(\boldsymbol{\Delta}_{\pi})|\mathcal{I}]$$

- System parameters $\mathcal{I}$ includes:
  1. Network topology $\mathcal{G}$
  2. Packet generation times $\{s_i\}_i$
  3. Packet arrival times at node 0 $\{a_{i0}\}_i$
  4. Buffer sizes $\{B_{ij}\}_{(i,j)\in\mathcal{L}}$

Results for General Service Time

Theorem 2: If packet transmission times are **arbitrary** given at each link, then for **all** $\mathcal{I}$ and $\pi \in \Pi_{npwc}$

$$[\boldsymbol{\Delta}_{\text{non-prmp-LGFS}}|\mathcal{I}] \leq_{\text{st}} [\boldsymbol{\Delta}_{\pi}|\mathcal{I}]$$

or equivalently, for **all** $\mathcal{I}$ and **non-decreasing functional** g

$$\mathbb{E}[g(\boldsymbol{\Delta}_{\text{non-prmp-LGFS}})|\mathcal{I}] \leq_{\text{st}} \min_{\pi \in \Pi_{npwc}} \mathbb{E}[g(\boldsymbol{\Delta}_{\pi})|\mathcal{I}]$$

- Π_{npwc}: Set of all **non-preemptive work-conserving** policies
- System parameters $\mathcal{I}$ includes:
  1. Network topology $\mathcal{G}$
  2. Packet generation times $\{s_i\}_i$
  3. Packet arrival times at node 0 $\{a_{i0}\}_i$
  4. Buffer sizes $\{B_{ij}\}_{(i,j)\in\mathcal{L}}$

These are the **first age optimality results** for multi-hop networks.

Proof idea

Step 1: System state process of policy π: $\{\mathbf{U}_\pi(t), t \in [0, \infty)\}$

$$\mathbf{U}_\pi(t) = (U_{0,\pi}(t), U_{2,\pi}(t), \ldots, U_{N-1,\pi}(t))$$

$U_{j,\pi}(t) = \max\{s_i : a_{ij} \le t\}$ **:** The generation time of the freshest packet that has arrived at node j at time t

Step 2: Coupling argument

Departure instants at each link are the same under all policies

Step 3: Use sample path argument to show that

$$\{\mathbf{U}_{\text{our policy}}(t), t \in [0, \infty)\} \ge \{\mathbf{U}_\pi(t), t \in [0, \infty)\}$$

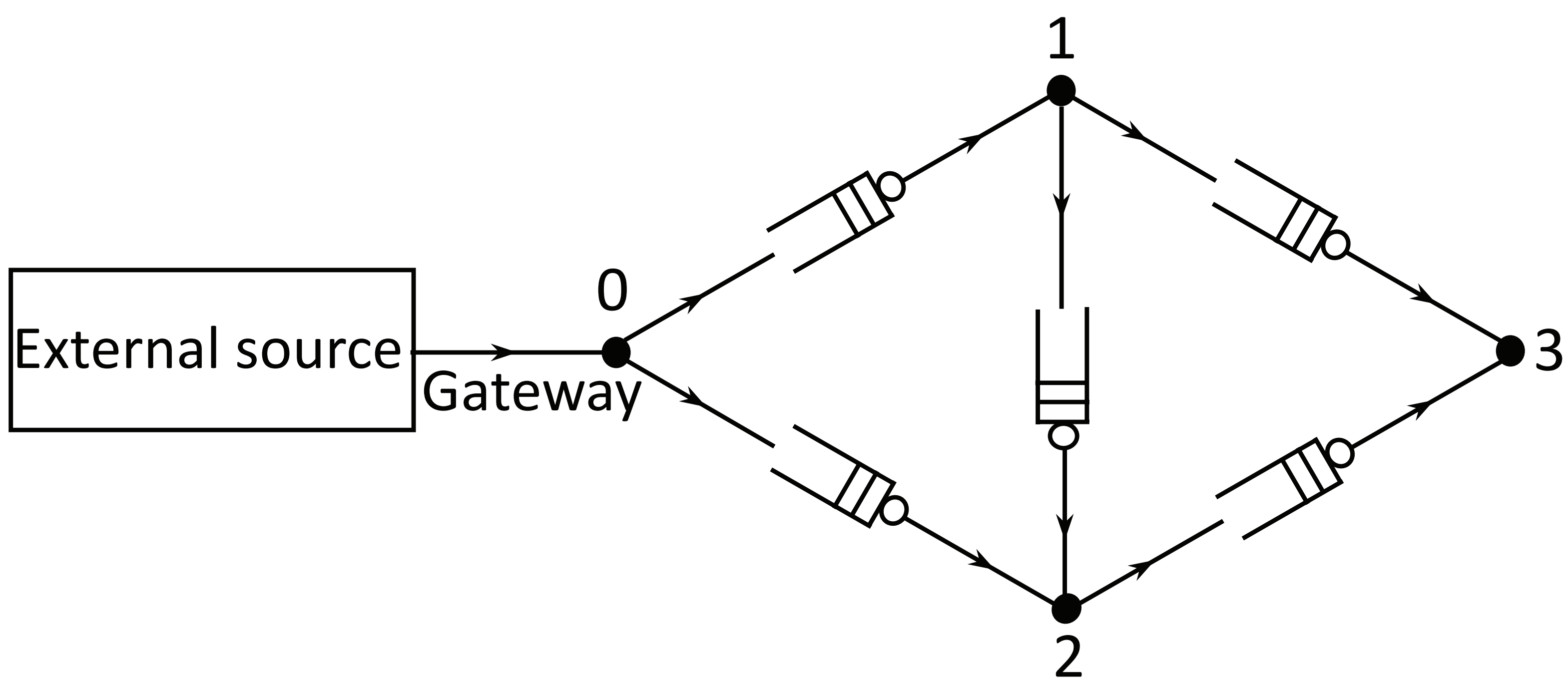
$\Downarrow$

$\{\mathbf{\Delta}(t), t \in [0, \infty)\}$ is **minimized** under our policies in **stochastic ordering sense**

$\Downarrow$

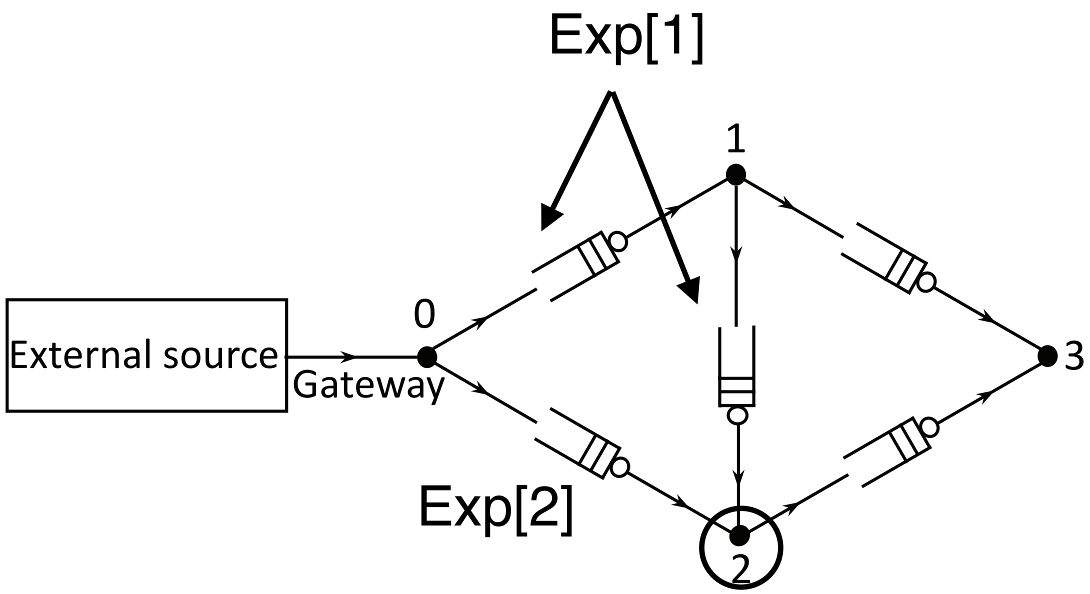
$\mathbb{E}[g(\mathbf{\Delta})]$ is **minimized** under our policies

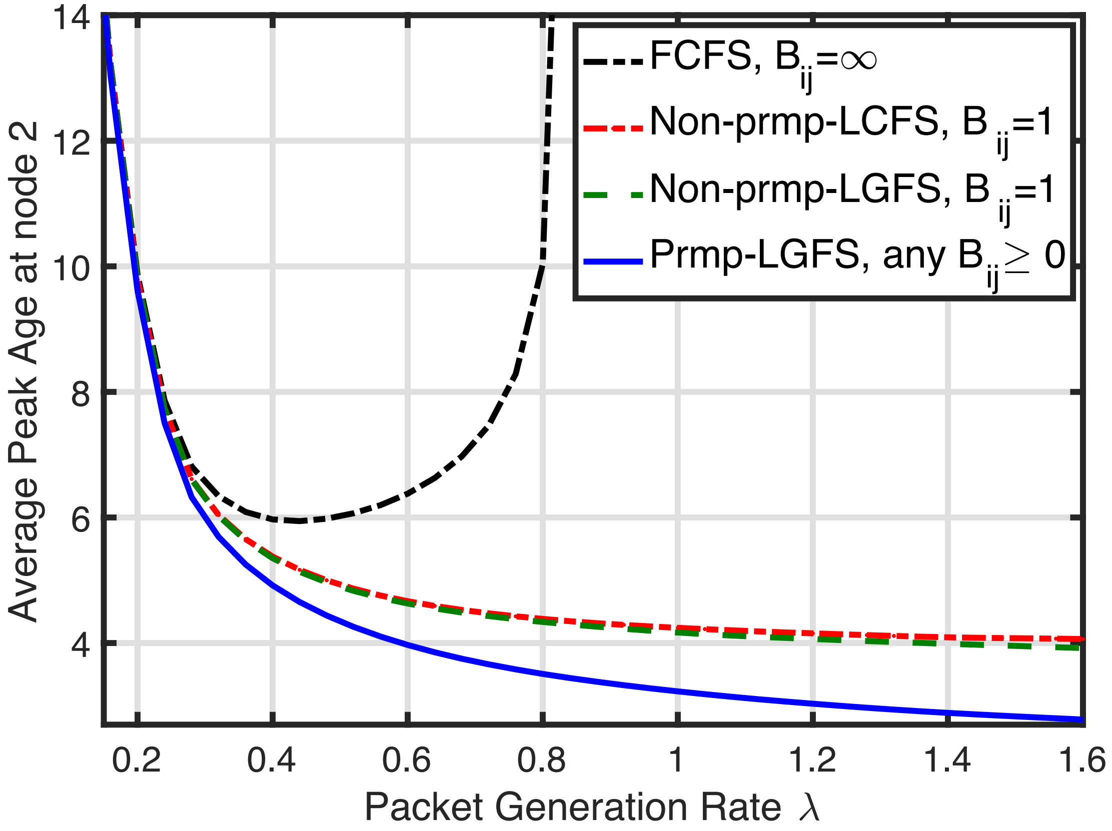
Simulation Result



- Inter-generation times: *i.i.d.* **Erlang-2 distribution**
- $(a_{i0} - s_i)$ is modeled to be either **1** or **100** with **equal probability**

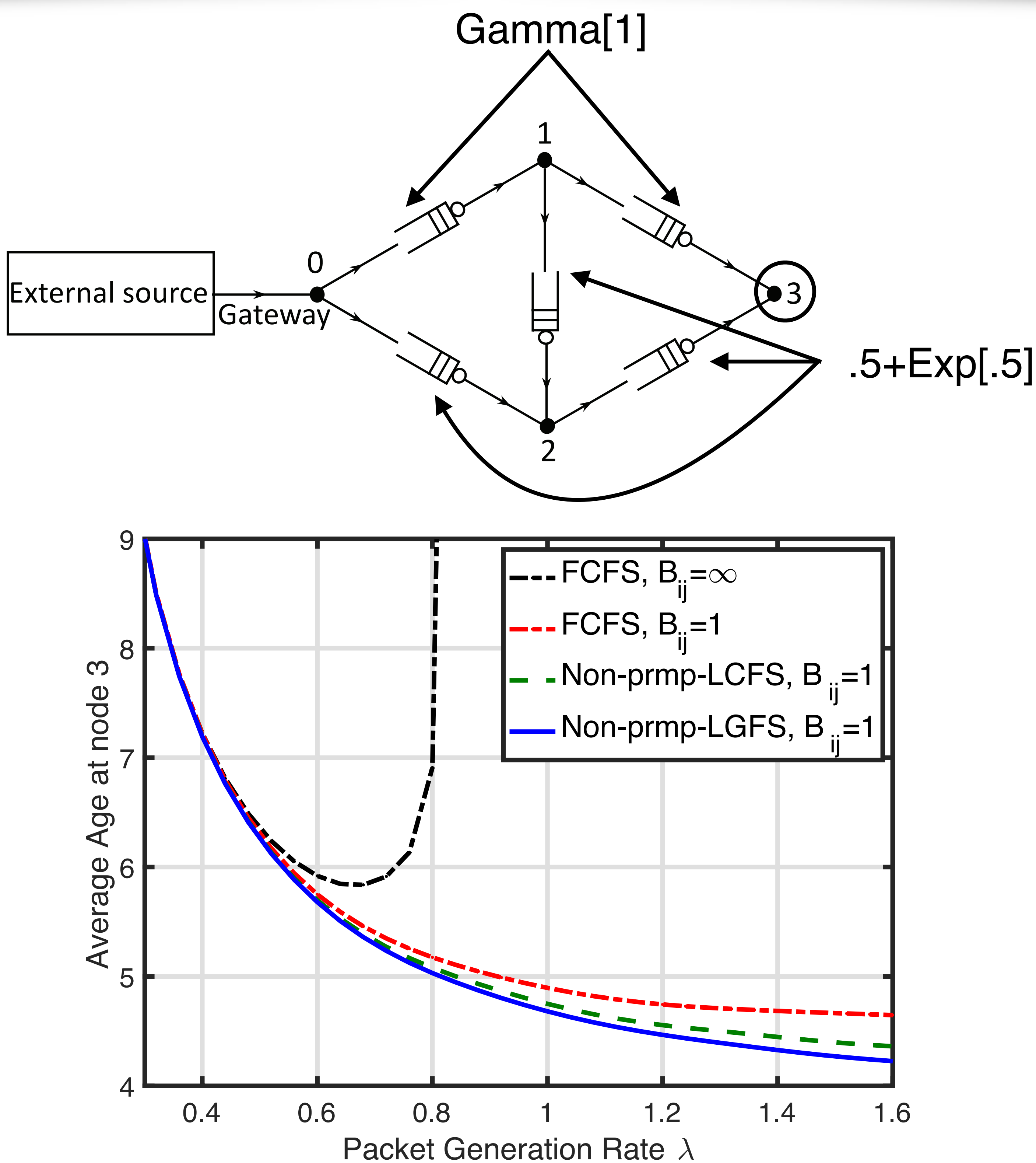
Simulation for Exponential Service Time





Observations: Preemptive LGFS **outperforms** all other policies.

Simulation for General Service Time



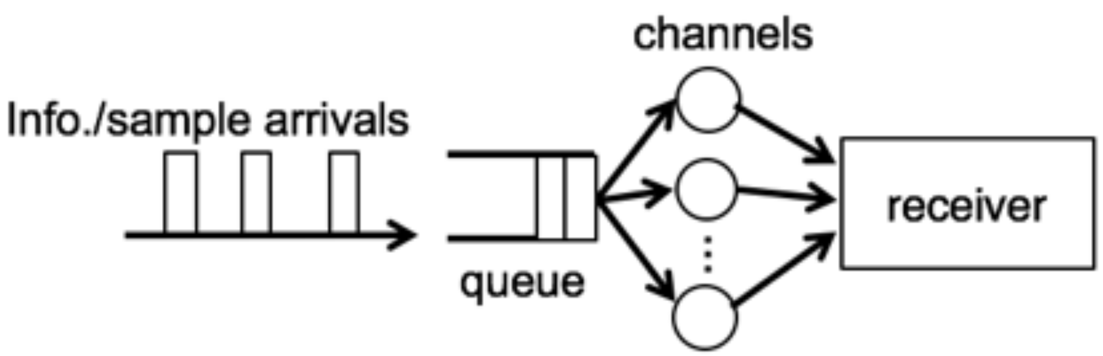
Observations: Non-preemptive LGFS **outperforms** all other non-preemptive work-conserving policies

Extension to Non-exponential Service Time

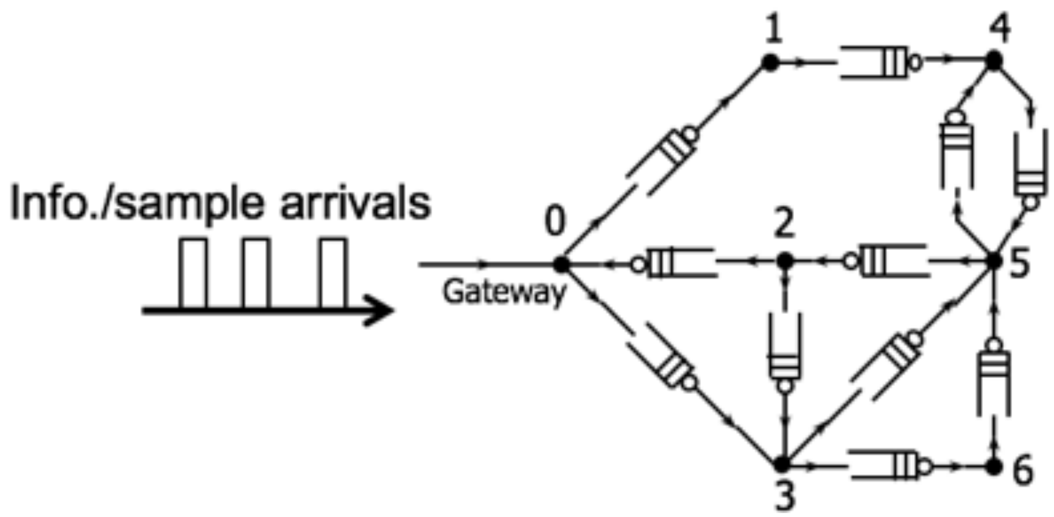
- **New-Better-than-Used (NBU) distributions**

(e.g. geometric, gamma, exponential, negative binomial distribution, etc.)

Model 1: Multi-channel network



Model 2: Multihop network



Thm: Suppose that the packet service times are NBU, then for all $\mathcal{I}$

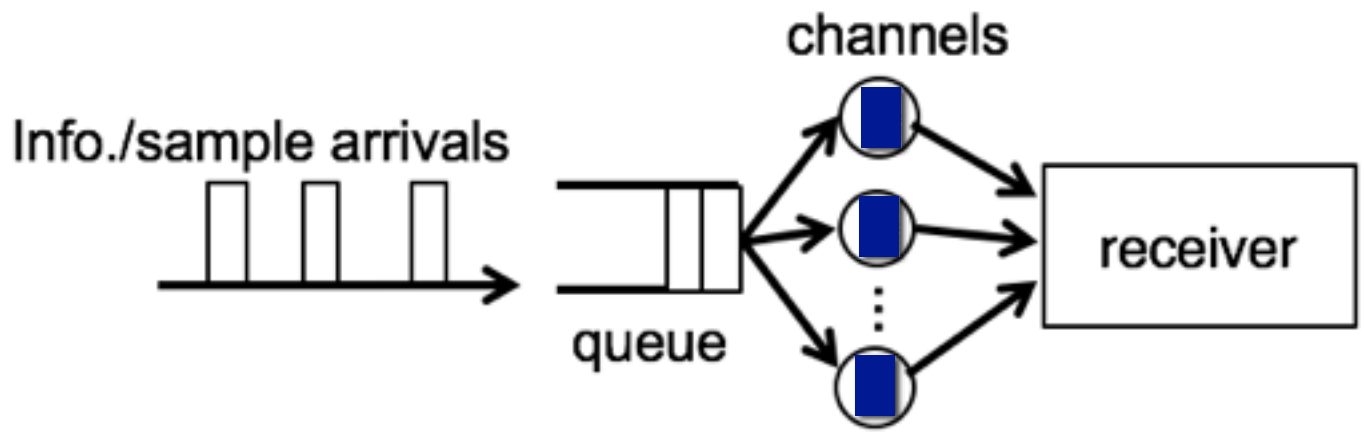
$$\min_{\pi\in\Pi}\mathbb{E}[\bar{\Delta}_\pi|\mathcal{I}] \leq \mathbb{E}[\bar{\Delta}_{\text{non-prmp-LGFS}}|\mathcal{I}] \leq \min_{\pi\in\Pi}\mathbb{E}[\bar{\Delta}_\pi|\mathcal{I}] + \mathbb{E}[X]$$

Thm: Suppose that the packet service times are NBU, then for all $\mathcal{I}$

$$\min_{\pi\in\Pi}\mathbb{E}[\bar{\boldsymbol{\Delta}}_\pi|\mathcal{I}] \leq \mathbb{E}[\bar{\boldsymbol{\Delta}}_{\text{non-prmp-LGFS}}|\mathcal{I}] \leq 3\min_{\pi\in\Pi}\mathbb{E}[\bar{\boldsymbol{\Delta}}_\pi|\mathcal{I}]$$

Where $\bar{\Delta}_\pi = \liminf_{T\to\infty}\frac{\int_0^T \Delta_\pi(t)dt}{T}$, $\mathbb{E}[X]$: Mean service time

Packet Replication may improve Age Performance



Packet replication technique

- Replication **worsens** the **Throughput** & **delay** performance **for NBU**:

[Sun, Koksal, Shroff'16]

Thm: Suppose that the packet service times are NBU, then for all $\mathcal{I}$

$$\min_{\pi\in\Pi}\mathbb{E}[\bar{\Delta}_{\pi}|\mathcal{I}] \leq \mathbb{E}[\bar{\Delta}_{\text{non-prmp-LGFS-R}}|\mathcal{I}] \leq \min_{\pi\in\Pi}\mathbb{E}[\bar{\Delta}_{\pi}|\mathcal{I}] + \mathbb{E}[X]$$

non-prmp-LGFS-R: Non-preemptive LGFS with replication policy

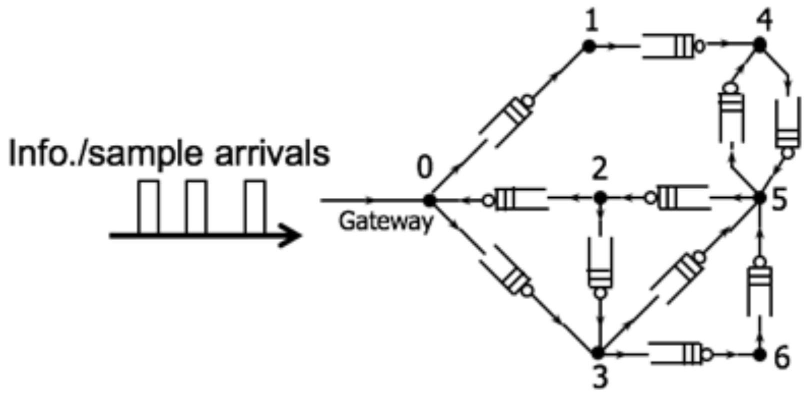
- **Replication helps in delivering fresh packets as soon as possible**
- **Replication exploits the diversity provided by multiple servers**

Summary

Most general age metric model:

- Any non-decreasing functional of the age processes of all nodes **(most general)**

System settings: **arbitrary** network topology, **arbitrary** packet generation & arrival processes



Contribution:

1. For **exponential** service times, **prmp-LGFS** is **age-optimal** among all causal policies
2. For **general** service times, **non-prmp-LGFS** is **age-optimal** among all non-preemptive work-conserving policies.
3. For **NBU** service times, **non-prmp-LGFS** is within three times of the optimum avg. age

Multi-channel single hop network:

1. For **NBU** service times, **non-prmp-LGFS** is within two times of the **optimum avg. age**
2. For **NBU** service times, **non-prmp-LGFS-R** is within two times of the **optimum avg. age**


Thank You!