ACM MobiHoc 2019

Age-optimal Sampling and Transmission Scheduling in Multi-Source Systems

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Joint work with Yin Sun[*], Sastry Kompella[§], Ness B. Shroff[‡]

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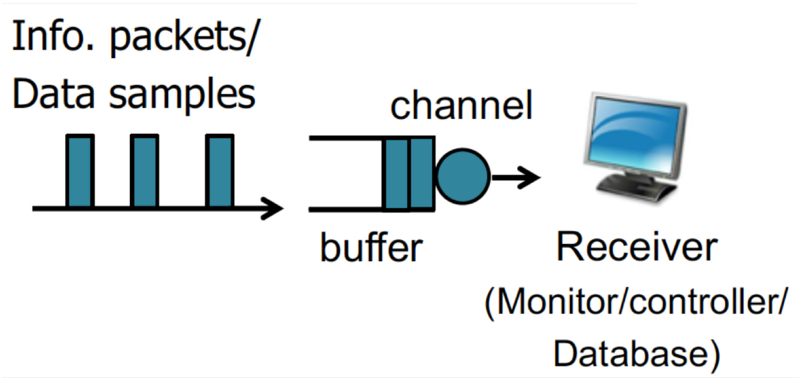
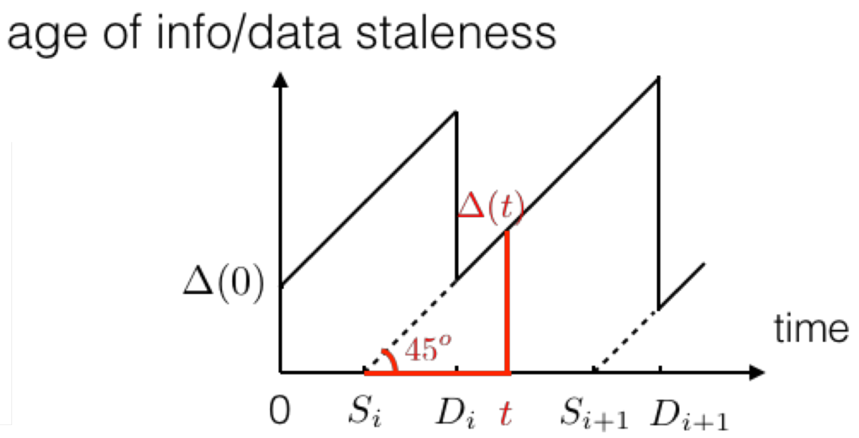
What is the Age of Information?





- In real-time applications, fresh data is more important than stale data
  - E.g., Autonomous vehicles, wireless sensor networks,…

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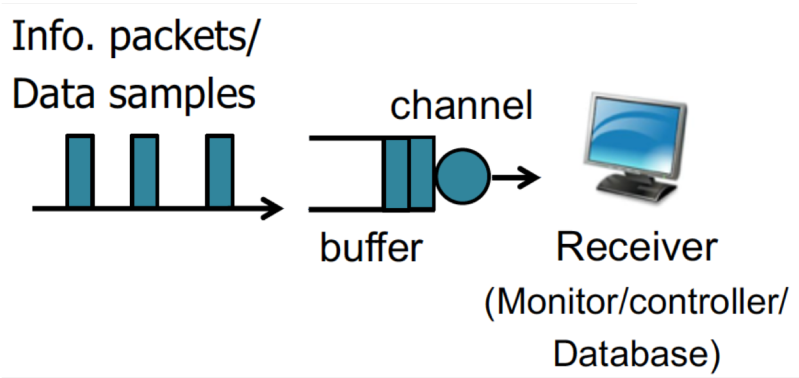
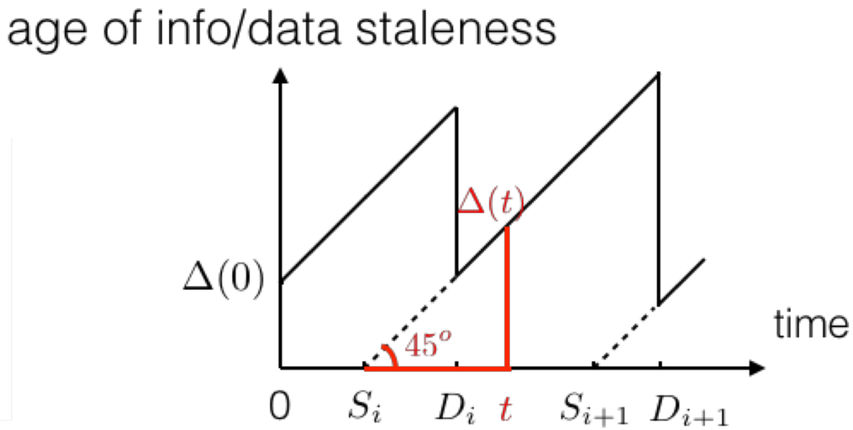




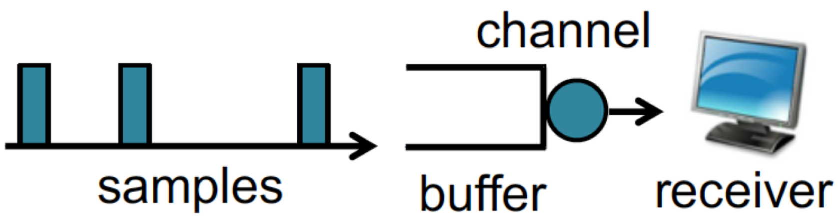
Definition: at any time t, **the age-of-information (AoI)** $\Delta(t)$ is the "age" of the freshest sample available at the destination before time t

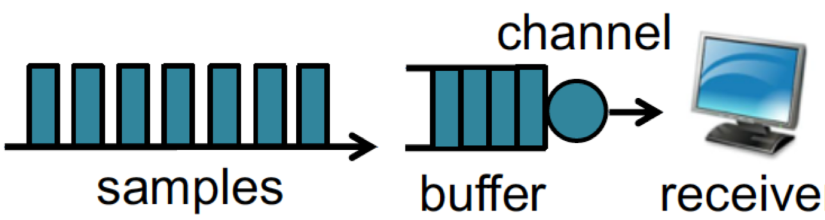
- If sample i is generated at S_i and delivered at D_i

$$\Delta(t) = t - \max\{S_i : D_i \leq t\}$$

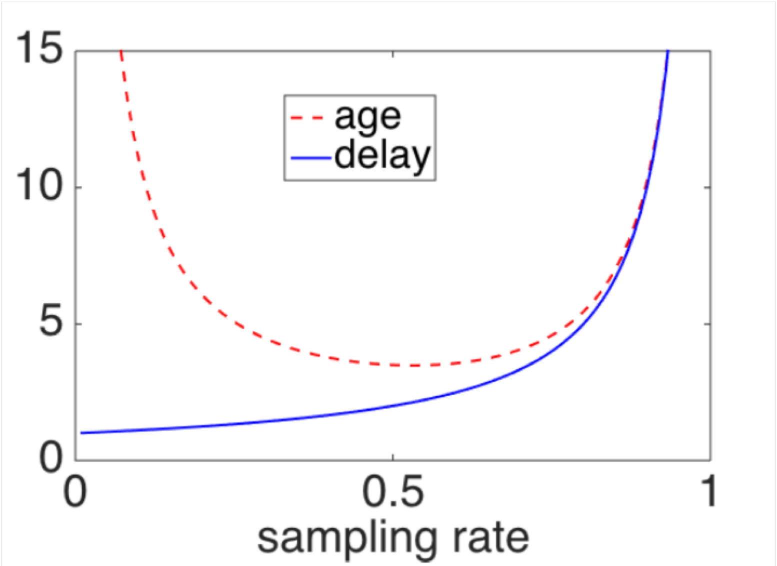
- Age grows linearly, and drops upon new sample delivered

Difference between Delay & Age





- Low sampling rate
  - Empty buffer ➡ Low delay
  - Infrequent updates ➡ High age
- High sampling rate
  - Full buffer ➡ High delay
  - Long waiting time ➡ High age



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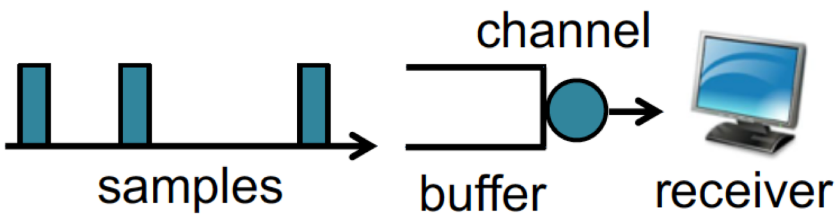
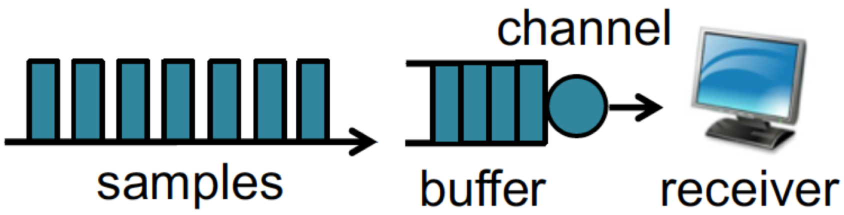
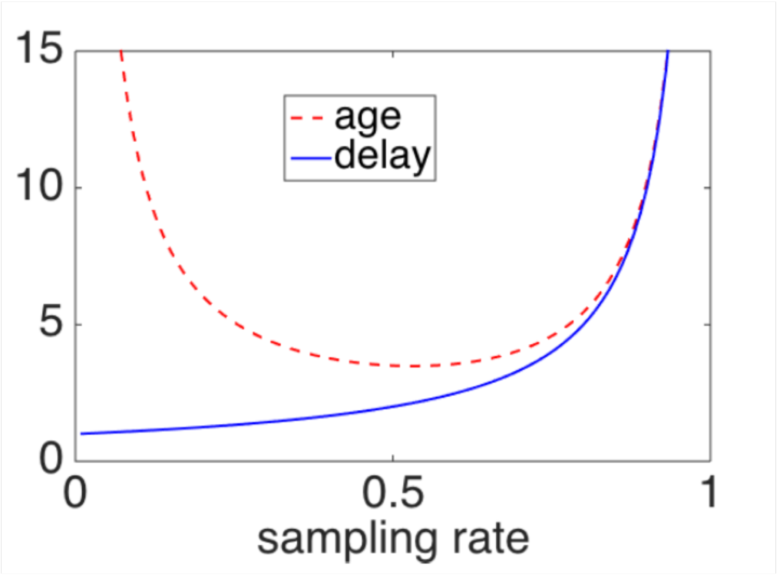
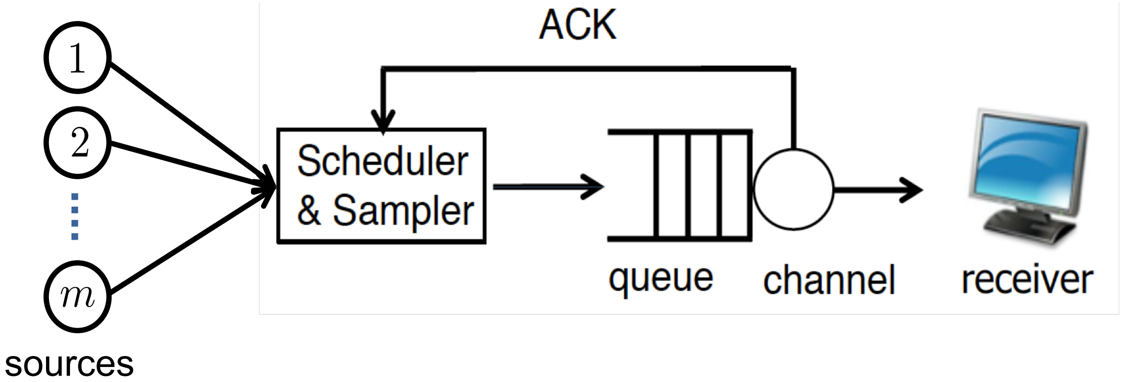




- Delay grows linearly wrt queue length ⇒ Little's law
- Age is not monotonic wrt queue length

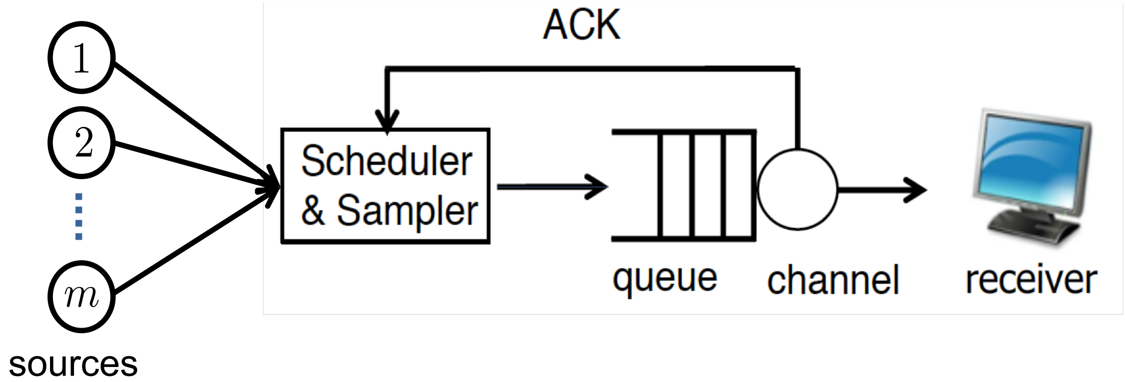


Our System Model



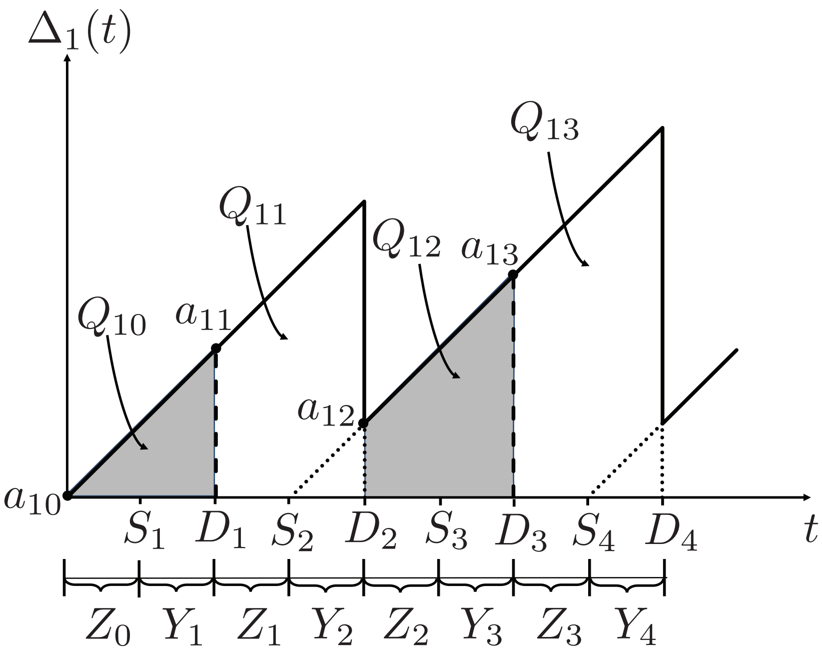
- Information update system with m sources
- **Channel:** FIFO queue with *i.i.d. service times*
- One source can communicate at a time
- **Scheduler:** Specifies the transmission order of the sources

Our System Model



- Controllable sample generation times
- Sample i is generated at S_i, with service time Y_i, and is delivered at D_i
  - $Y_i \geq 0$ can be any discrete random variable (*i.i.d.* & bounded)
- **Feedback:** Instantaneous Ack upon sample reception
- **Trick:** Only take sample when the server is idle, i.e., $S_{i+1} \geq D_i$
- Z_i: The waiting time after the delivery of packet i at D_i
- **Sampler:** Controls $(S_1, S_2, \ldots)$, or equivalently $(Z_0, Z_1, \ldots)$

Why Waiting Times?



- Multiple sources network
- S_2 and S_4 are generated from Source 1

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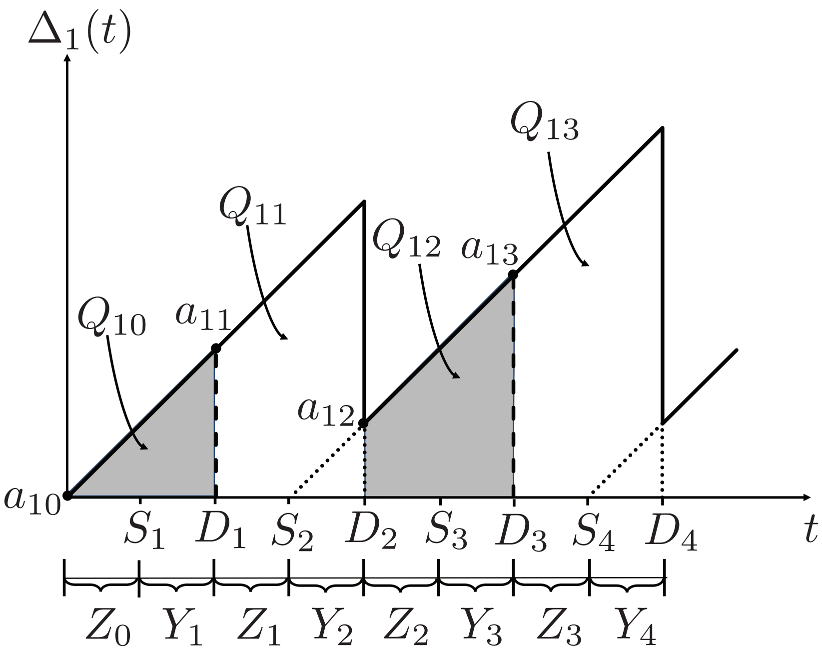

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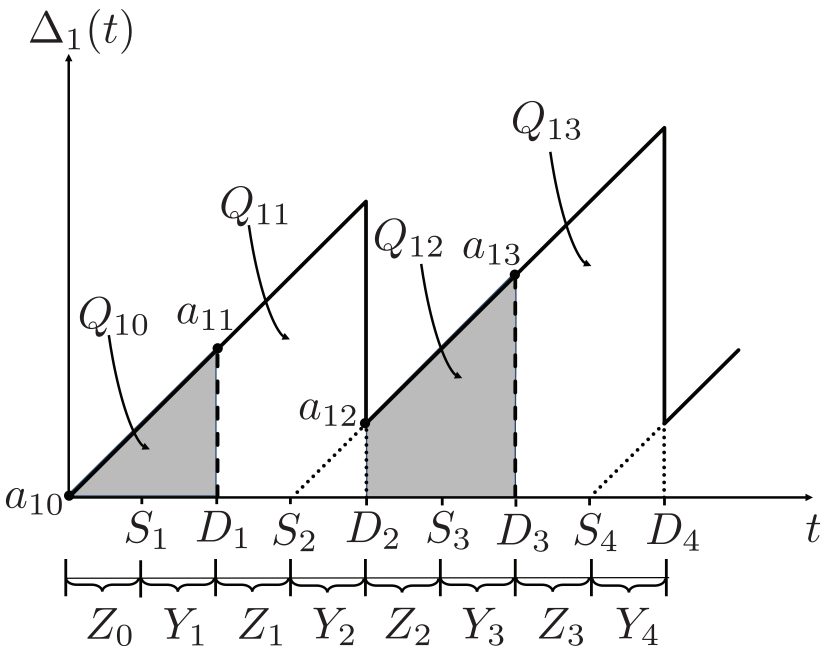


- Multiple sources network
- S_2 and S_4 are generated from Source 1

Why do we need to impose waiting times?

- **Natural choice: Zero-wait policy:**
  - Generate a sample as soon as the channel is idle ($S_{i+1} = D_i$)
- **Zero-wait** is NOT always Age-optimal!

Example: Zero-Wait is Not always Age-optimal

Example: Single source NW, channel transmission time = 0 or 2 with Prob. 0.5

0, 0, 2, 0, 2, 2, 0, 2, 0, 0, …

Zero-wait policy:

- Samples 1 & 2 are generated at the same time ➡ Sample 2 carries no information

Wasted Resources, Can we do better?

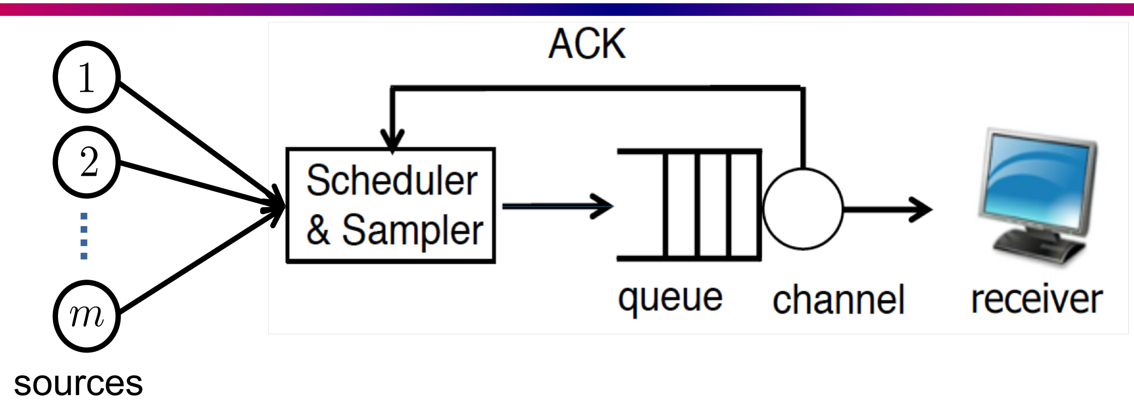
ϵ-wait policy:

- Wait for ϵ sec., if the previous sample has zero service time
- Don't wait otherwise.

Average age: $\bar{\Delta}(\epsilon) = (\epsilon^2 + 2\epsilon + 8)/(4 + 2\epsilon)$

- Zero-wait: $\bar{\Delta}(0) = 2$, ϵ-wait: $\bar{\Delta}(0.5) = 1.85$

Problem Formulation



- **Scheduler** π**:** Specifies the transmission order of the sources
- **Sampler** f**:** Controls $(S_1, S_2, \ldots)$, or equivalently $(Z_0, Z_1, \ldots)$
- **Challenge: Joint optimization** of **scheduler** and **sampler** for minimizing the total average age

$$\min_{\pi\in\Pi, f\in\mathcal{F}} \bar{\Delta}(f,\pi) = \min_{\pi\in\Pi, f\in\mathcal{F}} \limsup_{n\to\infty} \frac{\mathbb{E}\left[\sum_{l=1}^{m}\int_0^{D_n}\Delta_l(t)dt\right]}{\mathbb{E}[D_n]}$$

- Π: Set of causal schedulers $\mathcal{F}$:Set of causal samplers

Prior Works

Optimal scheduler for minimizing AoI in multi-source networks
(Time-slotted system)

Stochastic arrivals

- Wireless NW with interference:
[He, Yuan, Ephremides 2018]
- Broadcast NW:
[Hsu 2018]
[Hsu, Modiano, Duan 2017]
[Kadota, Sinha, Modiano 2018]
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Active sources

- Multiaccess channel:
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First to consider joint optimization of sampler + scheduler to minimize AoI:
- **Multisource networks**
- **Any discrete random transmission time**

Step 1: Separation Principle

Maximum Age First (MAF) scheduler:

- The source with the maximum age is served the first

[Li-Eryilmaz-Srikant'15, Kadota-Uysal-Singh-Modiano'16, Hsu-Modiano-Dua'17, Sun-Uysal-Kompella'18]

Proposition 1: For any given sampler $f \in \mathcal{F}$, MAF scheduler minimizes AoI compared to scheduling policies in Π, i.e.,

$$\bar{\Delta}(f, \pi_{\mathrm{MAF}}) \leq \bar{\Delta}(f, \pi) \ \forall f \in \mathcal{F}, \forall \pi \in \Pi$$

$\bar{\Delta}$: the total average age π_{MAF} : MAF scheduler

Proof idea: Stochastic ordering technique

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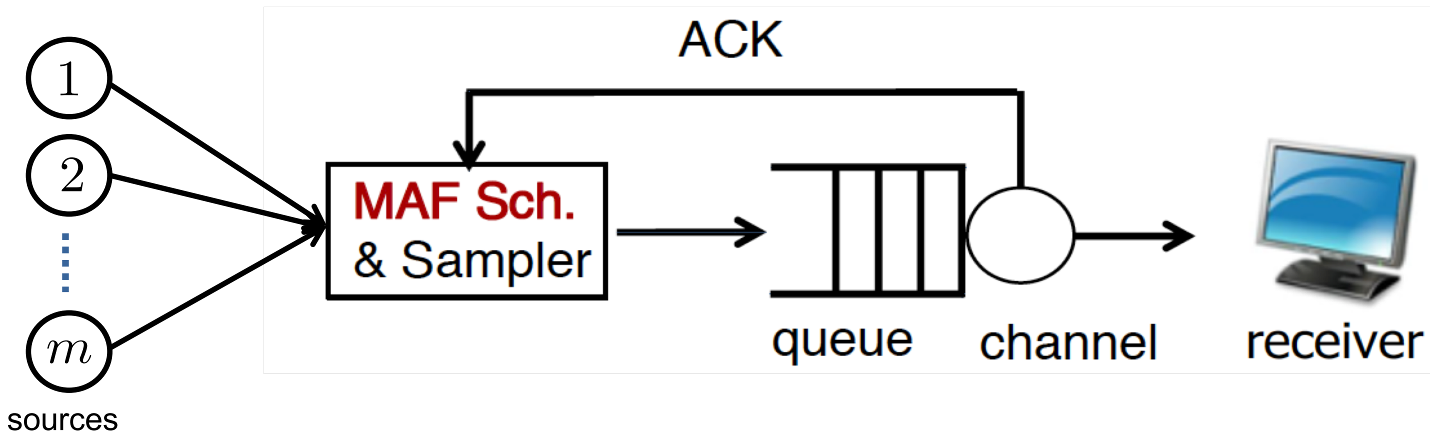
The scheduler and sampler can be designed **independently!**

Reduced Optimization Problem



- **Goal:**

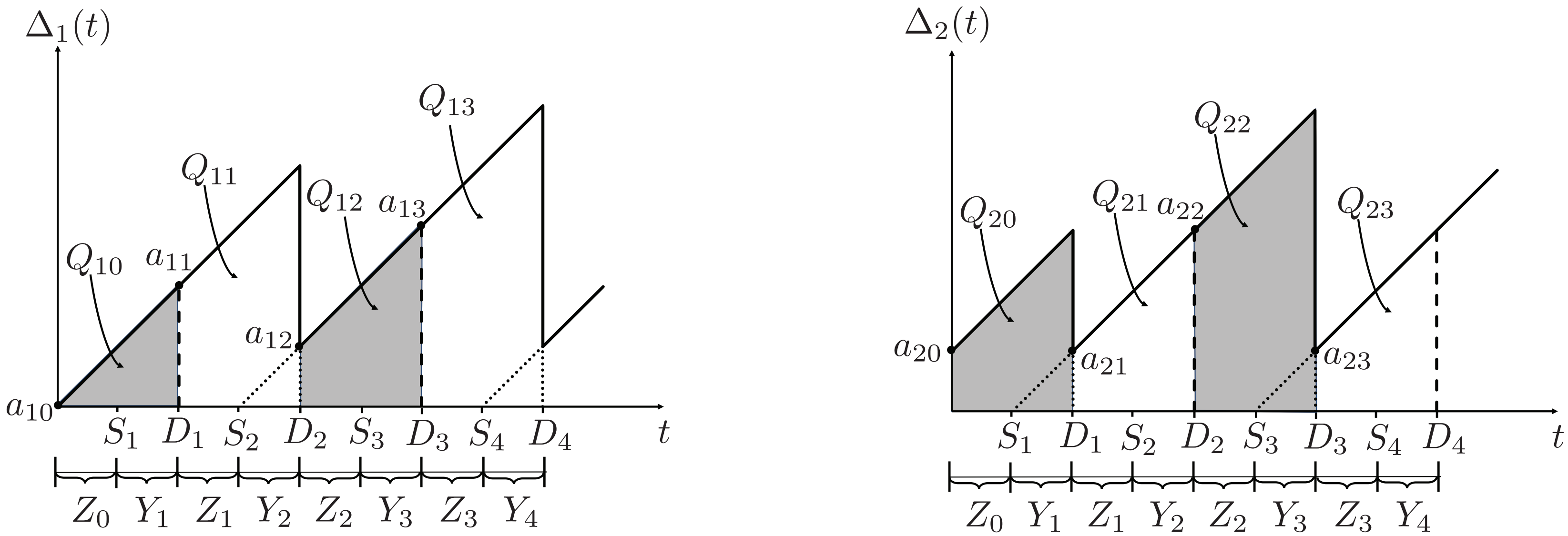
$$\min_{f \in \mathcal{F}} \bar{\Delta}(f, \pi_{MAF})$$

$$f \triangleq (Z_0, Z_1, \ldots)$$

$$\bar{\Delta}(f, \pi_{\text{MAF}}) = \limsup_{n \to \infty} \frac{\mathbb{E}[\sum_{l=1}^{m} \int_0^{D_n} \Delta_l(t) dt]}{\mathbb{E}[D_n]},$$

Reduced Optimization Problem

- Example: m=2



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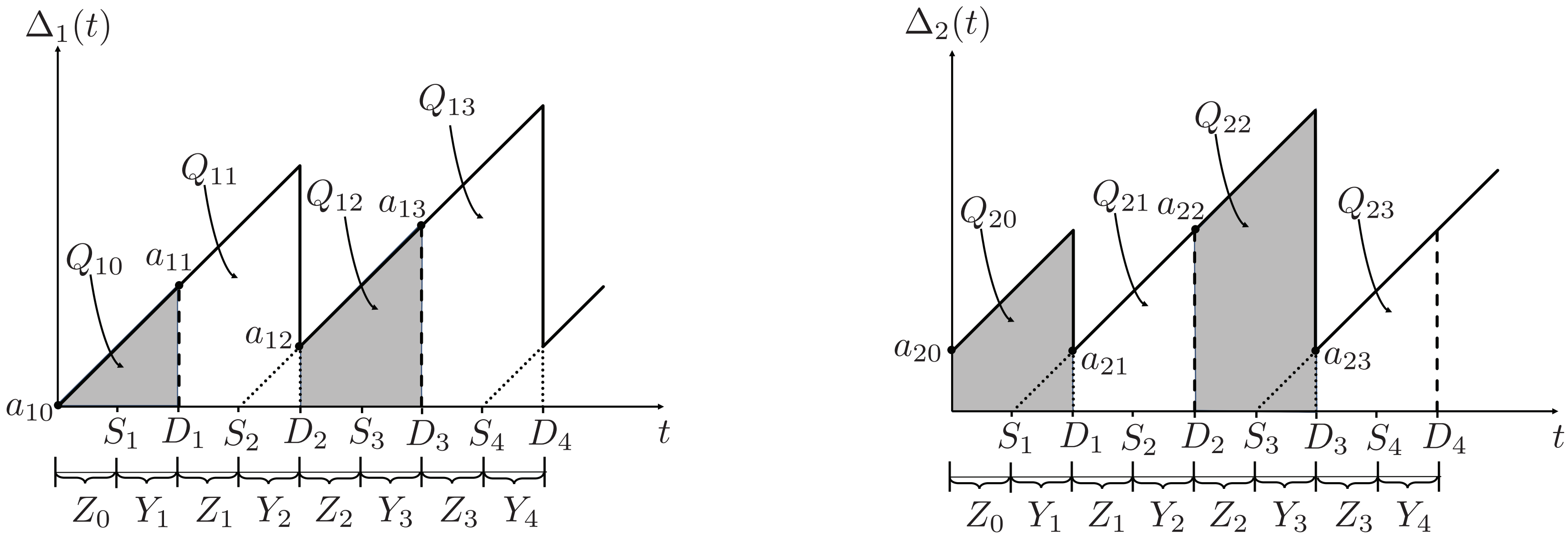

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Step 2: Equivalent Sampling Problem

- Problem **A**: $$\bar{\Delta}_{\text{opt}} = \min_{f \triangleq (Z_0, Z_1, \ldots)} \limsup_{n \to \infty} \frac{\sum_{i=0}^{n-1} \mathbb{E}[\sum_{l=1}^{m} Q_{li}]}{\sum_{i=0}^{n-1} \mathbb{E}[Z_i + Y_{i+1}]}$$

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- Problem **B**: $p(\beta) = \min_{f \triangleq (Z_0, Z_1, \ldots)} \limsup_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \mathbb{E}\left[\sum_{l=1}^{m} Q_{li} - \beta(Z_i + Y_{i+1})\right]$

Lemma:

1. If $p(\beta) = 0$, then the optimal samplers for Problems **A** and **B** are identical
2. $\bar{\Delta}_{\text{opt}} = \beta$ **iff** $p(\beta) = 0$

Algorithm:

1. Inner loop: Solve Problem **B**
2. Outer loop: Seek $\beta = \bar{\Delta}_{\text{opt}} \geq 0$, s.t. $p(\bar{\Delta}_{\text{opt}}) = 0$ (Bisection method)

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Dynamic Programming (DP)

- Problem **B**: $p(\beta) = \min_{f \triangleq (Z_0, Z_1, \ldots)} \limsup_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \mathbb{E}\left[\sum_{l=1}^{m} Q_{li} - \bar{\Delta}_{\text{opt}}(Z_i + Y_{i+1})\right]$
- Average Cost per stage DP problem: $\limsup_{n\to\infty} \frac{1}{n} \mathbb{E}\left[\sum_{i=o}^{n-1} C(s(i), Z_i)\right]$

State: $s(i) = (a_{[1]i}, \ldots, a_{[m]i})$ **,** $a_{[l]i}$: The l[th] largest age value at time D_i

State evolution: $a_{[m]i+1} = Y_{i+1}$

$$a_{[l]i+1} = a_{[l+1]i+1} + Z_i + Y_{i+1},\ l = 1, \ldots, m-1$$

Cost: $C(s(i), Z_i) = \mathbb{E}_{Y_{i+1}}\left[\sum_{l=1}^{m} Q_{li}(s(i), Z_i, Y_{i+1}) - \bar{\Delta}_{\text{opt}}(Z_i + Y_{i+1})\right]$

Solution of DP

Proposition 2: There exists a stationary deterministic policy that is average cost optimal and solves the following Bellman's equation:

$$\lambda + h(s) = \min_{z} \left[C(s,z) + \sum_{s'} \mathbb{P}_{ss'} h(s') \right]$$

λ : The optimal average cost

$h(s)$: Relative cost function

$\mathbb{P}_{ss'}$: Transition probability

Proof idea: Communicating MDP

- Relative value iteration (RVI)

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Proof idea: Communicating MDP

- Relative value iteration (RVI) $\xrightarrow{\text{Suffer}}$ Curse of dimensionality
- We need simplification

Threshold-Based Sampler

Proposition 3 (Threshold-based sampler): The optimal waiting time is **ZERO** for states whose $A_s \geq (\bar{\Delta}_{\text{opt}} - m\mathbb{E}[Y])$.

Algorithm 1: Threshold-based sampler based on RVI algorithm.

---

given $l = 0$, sufficiently large u, tolerance $\epsilon_1 > 0$, tolerance $\epsilon_2 > 0$;
while $u - l > \epsilon_1$ **do**
    $\beta = \frac{l+u}{2}$;
    $J(\mathbf{s}) = 0$, $h(\mathbf{s}) = 0$, $h_{\text{last}}(\mathbf{s}) = 0$ for all states $\mathbf{s} \in \mathcal{S}$;
    while $\max_{\mathbf{s} \in \mathcal{S}} |h(\mathbf{s}) - h_{last}(\mathbf{s})| > \epsilon_2$ **do**
        for each $\mathbf{s} \in \mathcal{S}$ **do**
            if $A_s \geq (\beta - m\mathbb{E}[Y])$ **then**
                $z_s^* = 0$;
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                $z_s^* = \text{argmin}_{z \in \mathcal{Z}} (A_s - \beta)(z + \mathbb{E}[Y]) + \frac{m}{2}(z^2 + 2z\mathbb{E}[Y] + \mathbb{E}\left[Y^2\right]) + \sum_{\mathbf{s}' \in \mathcal{S}} \mathbb{P}_{\mathbf{s}\mathbf{s}'}(z) h(\mathbf{s}')$;
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        end
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        $h(\mathbf{s}) = J(\mathbf{s}) - J(\mathbf{o})$;
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If test fails, use RVI

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  end
  if $J(\mathbf{o}) \geq 0$ **then**
    $u = \beta$;
  else
    $l = \beta$;
  end
end

Outer loop: Bisection method

Apply threshold test

If test fails, use RVI

Threshold-Based Sampler

Proposition 3 (Threshold-based sampler): The optimal waiting time is **ZERO** for states whose $A_s \geq (\bar{\Delta}_{\text{opt}} - m\mathbb{E}[Y])$.

As a result of Propositions 1, 2, and 3, we get

Theorem: The **MAF** scheduler and the **threshold-based** sampler are jointly optimal for minimizing the total average age.

Bellman's Equ. Approximation

- Towards a simpler solution:
- Bellman's equation: $$\lambda = \min_z \left[C(s,z) + \sum_{s'} \mathbb{P}_{ss'}(h(s') - h(s)) \right]$$

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After the substitution and taking the derivative

- Water-filling solution: $\hat{z}_s^\star = \left[th - \frac{A_s}{m} \right]^+$

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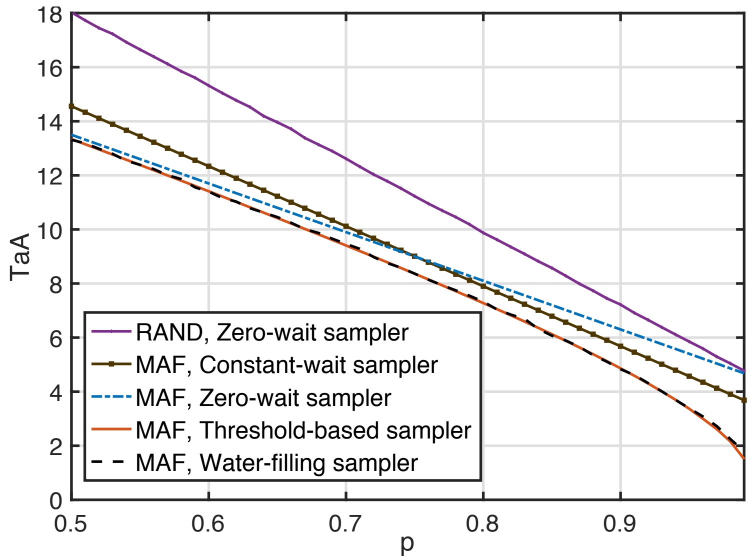
After the substitution and taking the derivative

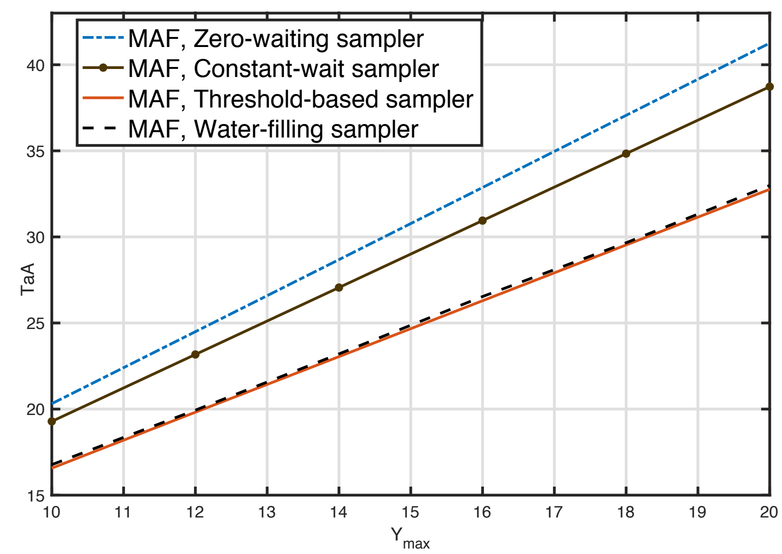
- Water-filling solution: $$\hat{z}_s^\star = \left[th - \frac{A_s}{m} \right]^+$$

Golden-section method to obtain optimal threshold

Simulation Results



3 sources, service=[0 3], prob=[p 1-p]



3 sources, service=[0 Y_{max}], prob=[.9 .1]

Observations:

1. MAF scheduler outperforms the RAND scheduler
2. With MAF, threshold-based outperforms zero-wait and constant-wait:
   - Zero-wait sampler is Not always optimal
   - Optimizing the scheduler is not enough
3. The performance of Water-filling and threshold-based are almost the same

Avg. Peak Age problem

- Much simpler problem

Theorem: The **MAF** scheduler and the **zero-wait** sampler are jointly optimal for minimizing the total average **peak** age.

- What minimizes avg peak AoI doesn't necessary minimize avg AoI

Summary & Future work

- Joint optimization of the scheduler and sampler for minimizing AoI.
- Separation principle: The scheduler and sampler can be designed independently
- MAF scheduler and Threshold-based sampler are jointly optimal for avg. AoI
- Water-filling sampler can approximate the threshold-based sampler
  - Simulations show that their performances are almost the same
- MAF scheduler and zero-wait sampler are jointly optimal for avg. peak AoI

- **Future work:**
  - **Symmetric non-linear age functional**
  - **Asymmetric non-linear age functional**

Q&A

Thanks